Introduction to Perturbative QCD

Lecture 3

Jianwei Qiu
Iowa State University/Argonne National Laboratory

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Outline for Lecture 3

- Cross section with one identified hadron
  – Lepton-hadron deeply inelastic scattering (DIS)
- Factorization for IR sensitive cross sections
- What is the predictive power of pQCD?
- DGLAP evolution equation
- Parton distribution functions

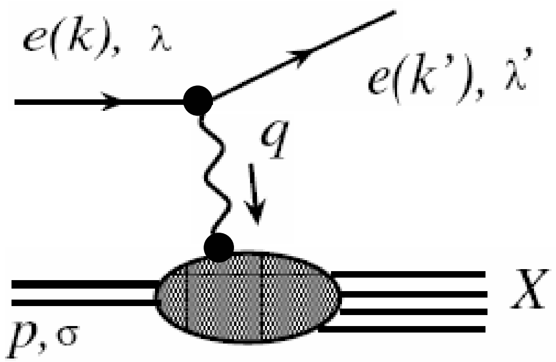
Excellent resource – CTEQ summer school website

http://www.phys.psu.edu/~cteq

Deep inelastic scattering

- **Recall:**

$$E'\frac{d\sigma^{\text{DIS}}}{d^3k'} = \frac{1}{2s}\left(\frac{1}{Q^2}\right)^2 L^{\mu\nu}(k,k')W_{\mu\nu}(q,p)$$



- **Hadronic tensor:**

$$W_{\mu\nu}(q,p,S) = \frac{1}{4\pi}\int d^4z\, e^{iq\cdot z}\left\langle p,S\left|J_\mu^\dagger(z)J_\nu(0)\right|p,S\right\rangle$$

$$W_{\mu\nu} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)F_1\left(x_B,Q^2\right) + \frac{1}{p\cdot q}\left(p_\mu - q_\mu\frac{p\cdot q}{q^2}\right)\left(p_\nu - q_\nu\frac{p\cdot q}{q^2}\right)F_2\left(x_B,Q^2\right)$$

$$+iM_p\varepsilon^{\mu\nu\rho\sigma}q_\rho\left[\frac{S_\sigma}{p.q}g_1\left(x_B,Q^2\right) + \frac{(p.q)S_\sigma - (S.q)p_\sigma}{(p.q)^2}g_2\left(x_B,Q^2\right)\right]$$

- **Structure functions – infrared sensitive:**

$$F_1\left(x_B,Q^2\right), F_2\left(x_B,Q^2\right), g_1\left(x_B,Q^2\right), g_2\left(x_B,Q^2\right)$$

Perturbative QCD Factorization

- Cross sections **with identified hadrons** are infrared sensitive and non-perturbative

  Typical hadronic scale: $1/R \sim 1\ \text{fm}^{-1} \sim \Lambda_{QCD}$

  Energy exchange in hard collisions: $Q >> \Lambda_{QCD}$

  ⇨ pQCD works at $\alpha_s(Q)$, but not at $\alpha_s(1/R)$

- PQCD can be useful **iff quantum interference** between perturbative and nonperturbative scales can be **neglected**

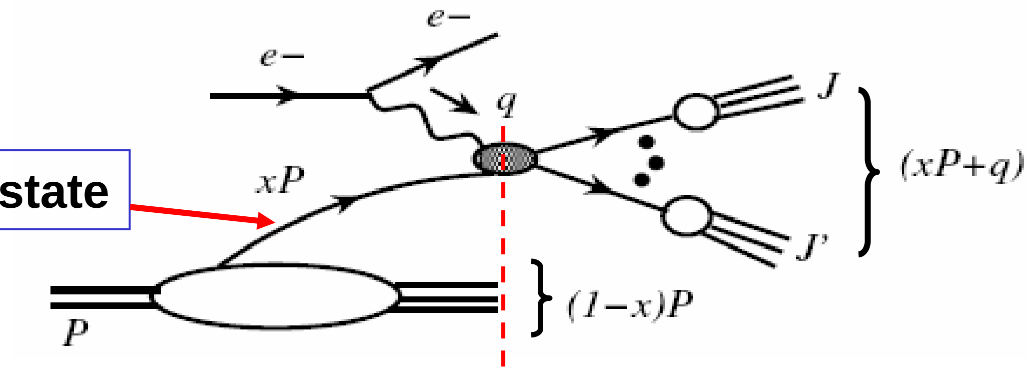
$$\sigma_{\text{phy}}(Q,1/R) \sim \hat{\sigma}(Q) \otimes \varphi(1/R) + O(1/QR)$$

Measured: σ_{phy}; Short-distance: $\hat{\sigma}(Q)$; Long-distance: $\varphi(1/R)$; Power corrections: $O(1/QR)$

Factorization ⟺ needs a "long-lived" parton state

Picture of factorization in DIS

❑ Time evolution:

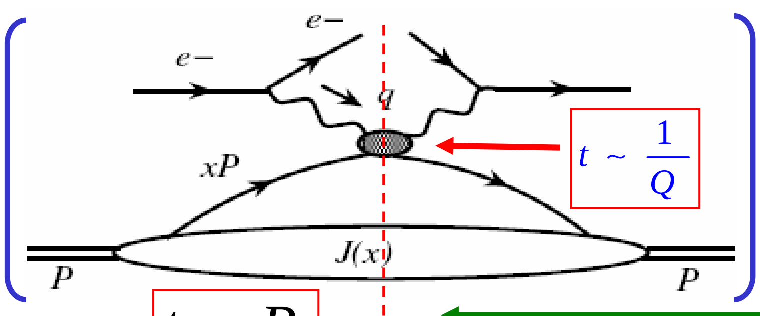


Long-lived parton state

Time: "Past" **Now** "Future"

❑ Unitarity – summing over all hard jets:

$$\sigma_{\text{tot}}^{\text{DIS}} \propto \text{Im}$$

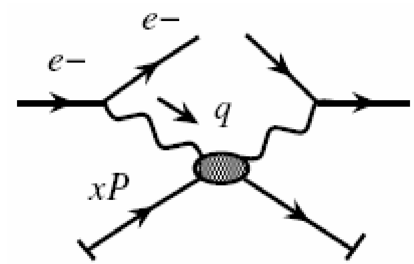


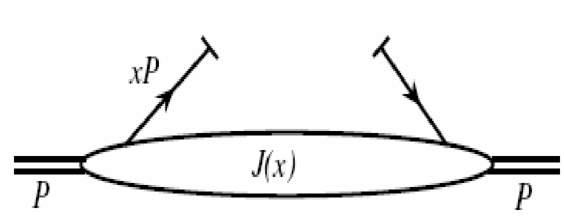
$t \sim R$

Not IR safe

Interaction between the "past" and "now" are suppressed!

Factorization in DIS

$\sigma_{\text{tot}}^{\text{DIS}} \sim$ 

$\otimes$ 

$+\, O\left(\frac{1}{QR}\right)$

Now | Past | Connection

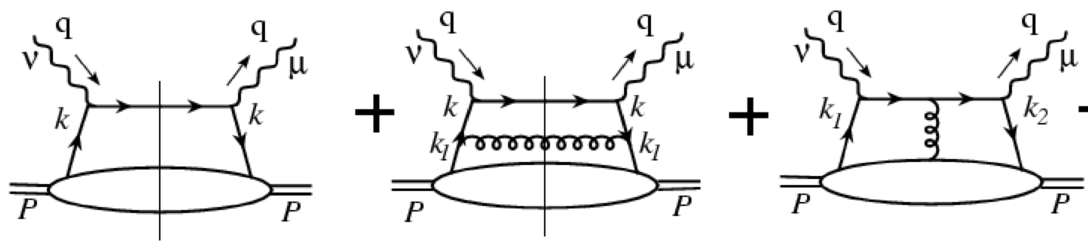
Predictive power of pQCD

- ❖ short-distance and long-distance are separately gauge invariant
- ❖ short-distance part is Infrared-Safe, and calculable
- ❖ long-distance part can be defined to be Universal

Long-lived parton states

- **Feynman diagram representation:**

$W^{\mu\nu} \propto$



$+ \ldots$

- **Perturbative pinched poles:**

$$\int d^4k \; H(Q,k)\left(\frac{1}{k^2+i\varepsilon}\right)\left(\frac{1}{k^2-i\varepsilon}\right) T(k,\frac{1}{r_0}) \;\Rightarrow\; \infty \;\text{ perturbatively}$$

- **Perturbative factorization:**

$$k^\mu = xp^\mu + \frac{k^2+k_T^2}{2xp\cdot n}n^\mu + k_T^\mu$$

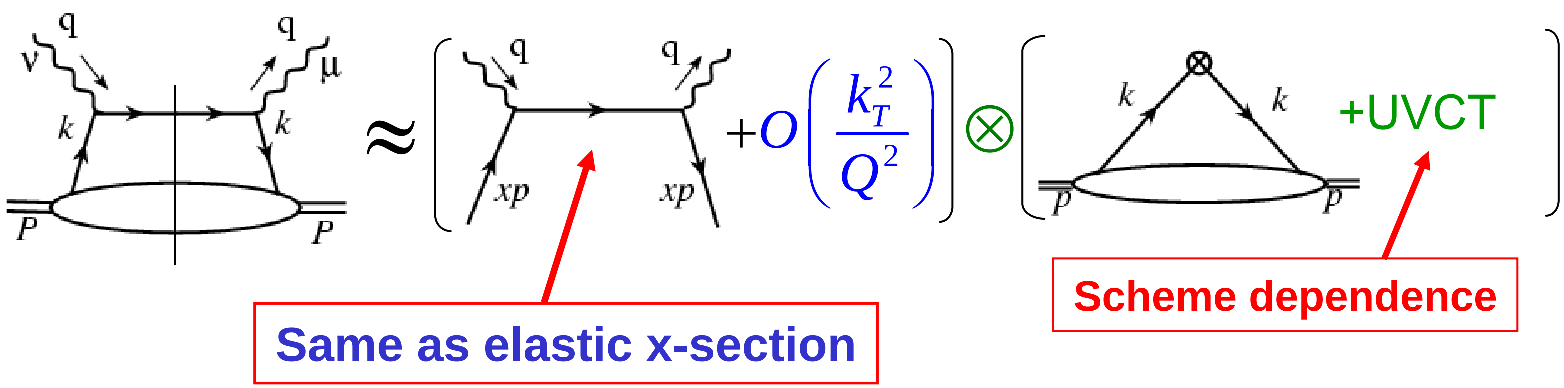
$$\int \frac{dx}{x} d^2k_T \; H(Q,k^2=0) \int dk^2 \left(\frac{1}{k^2+i\varepsilon}\right)\left(\frac{1}{k^2-i\varepsilon}\right) T(k,\frac{1}{r_0})$$

Nonperturbative matrix element

Short-distance

Collinear factorization

- **Collinear approximation, if** $Q \sim xp \cdot n \gg k_T, \sqrt{k^2}$



Parton's transverse momentum is integrated into parton distributions, and provides a scale of power corrections

- **DIS limit:** $\nu, Q^2 \to \infty$, while x_B fixed

⟹ **Feynman's parton model and Bjorken scaling**

$$F_2(x_B, Q^2) = x_B \sum_f e_f^2 \, \varphi_f(x_B) + O(\alpha_s) + O\left(\frac{\Lambda_{\rm QCD}^2}{Q^2}\right)$$

Necessary condition for factorization

"Any uncanceled long-distance divergence of a partonic scattering cross section has to be process-independent"

On hadron state: $\sigma_{H}(Q,1/R) \sim \sum_{a} \hat{\sigma}_{a}(Q) \otimes \varphi_{a/H}(1/R) + O(1/QR)$

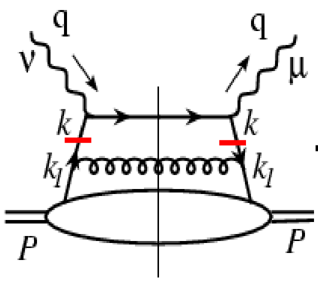
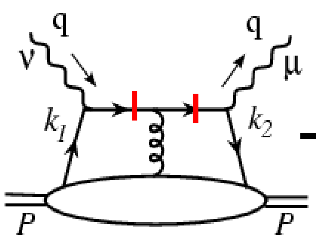
On parton state: $\sigma_{p}(Q,1/R) \sim \sum_{a} \hat{\sigma}_{a}(Q) \otimes \varphi_{a/p}(1/R) + O(1/QR)$

Process dependent partonic cross section (Feynman diagrams)

Process-independent Parton-level pdfs (Feynman diagrams)

Equal long-distance physics

Example:

$W_{\mu\nu}^{(1)} \propto$  + ...  + ...

All uncanceled divergences are absorbed into PDFs

Parton distribution functions (PDFs)

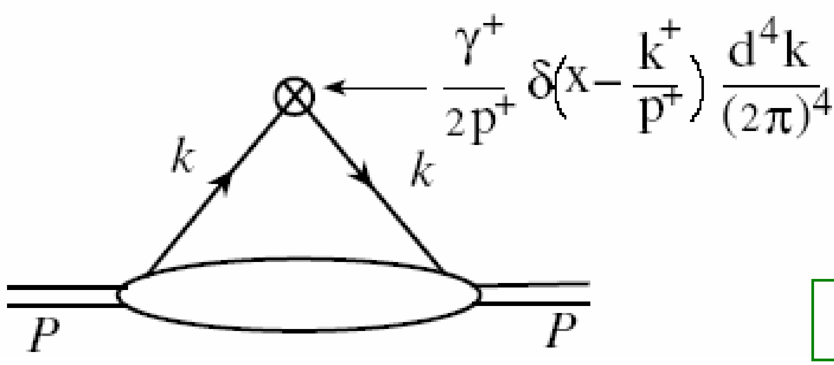
- **Predictive power of pQCD relies on the factorization and the universality of PDFs**

- **PDFs as matrix elements of two parton fields:**

  - quark distribution as an example,

$$\phi_{q/h}(x,\mu^2) = \int \frac{dy^-}{2\pi}\, e^{ixp^+y^-} \langle h(p)|\bar{\psi}_q(0)\, \frac{\gamma^+}{2} \mathcal{P} e^{-ig\int_0^{y^-} dw^- A^+(w^-)} \psi_q(y^-)|h(p)\rangle$$

  - corresponding diagram in momentum space,



+ UV CT

Gives the μ-dependence

  - $|h(p)\rangle$ can be a hadron, or a nucleus state, as well as a parton state

An instructive exercise for high orders

- **Consider a cross section:** $\sigma(Q^2, m^2) = \sigma_0\left[1+\alpha_s I + O(\alpha_s^2)\right]$
- **Leading quantum correction:** $I = \int_0^\infty dk^2 \, \frac{1}{k^2+m^2} \, \frac{Q^2}{Q^2+k^2}$
- **Analysis of the integral:**

$$I = \int_{k^2 \ll Q^2} dk^2 \, \frac{1}{k^2+m^2} + \int_{k^2 \sim Q^2} dk^2 \, \frac{1}{k^2} \, \frac{Q^2}{Q^2+k^2} + \mathcal{O}(m^2/Q^2)$$

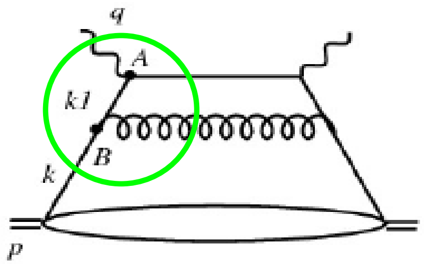
- **Result for the cross section:**

$$\sigma = \left(1 + \alpha_s \int_{k^2 \ll Q^2} dk^2 \, \frac{1}{k^2+m^2}\right)\left(1 + \alpha_s \int_{k^2 \sim Q^2} dk^2 \, \frac{1}{k^2} \, \frac{Q^2}{Q^2+k^2}\right) + \mathcal{O}(\alpha_s^2) + \mathcal{O}(m^2/Q^2)$$

$$\equiv f \times \hat{\sigma} + \mathcal{O}(\alpha_s^2) + \mathcal{O}(m^2/Q^2).$$

Scaling violation and factorization

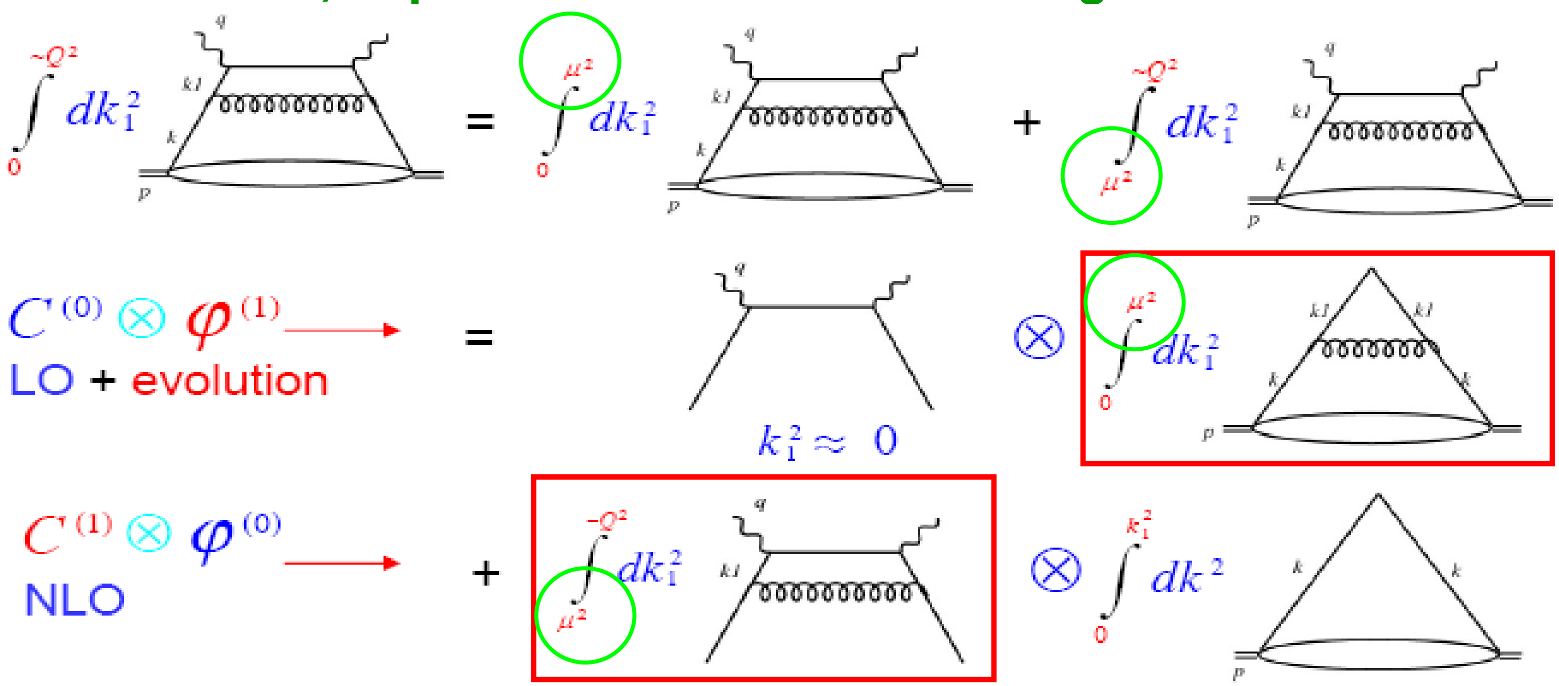
- **NLO partonic diagram to structure functions:**



$$\propto \int_0^{-Q^2} \frac{dk_1^2}{k_1^2} \quad \text{Dominated by} \quad \begin{cases} k_1^2 \sim 0 \\ t_{AB} \to \infty \end{cases}$$

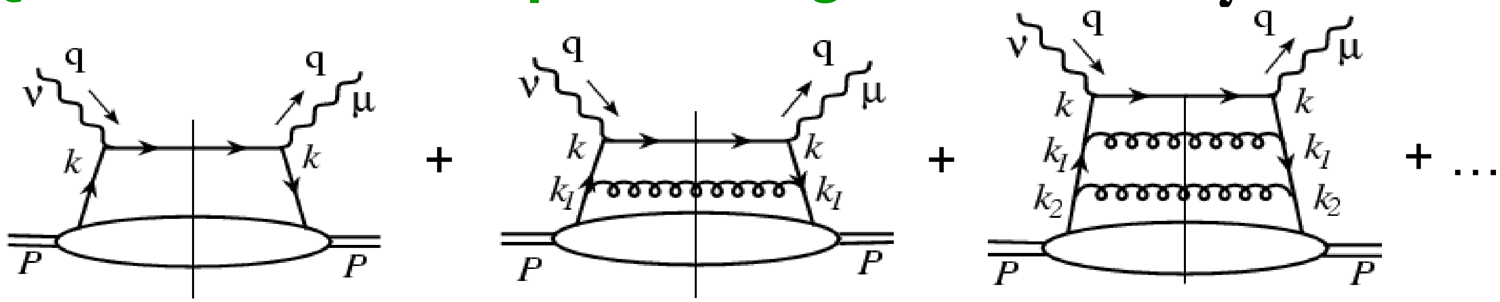
Diagram has both long- and short-distance physics

- **Factorization, separation of short- from long-distance:**

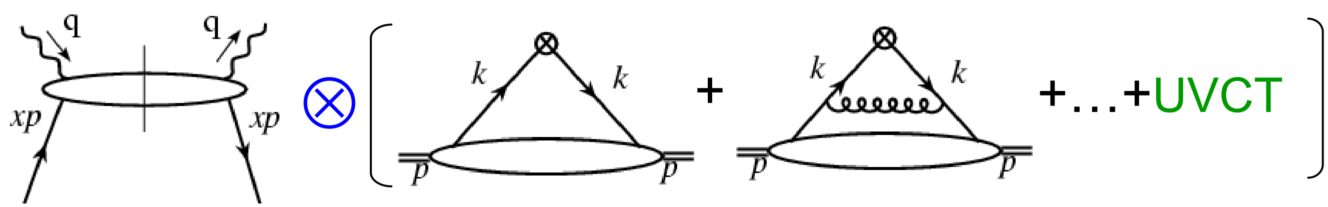


Leading power QCD formula

- ❑ QCD corrections: pinch singularities in $\int d^4 k_i$



- ❑ Logarithmic contributions into parton distributions



$$\Rightarrow F_2(x_B,Q^2) = \sum_f C_f\left(\frac{x_B}{x},\frac{Q^2}{\mu_F^2},\alpha_s\right)\otimes\varphi_f\left(x,\mu_F^2\right)+O\left(\frac{\Lambda_{\rm QCD}^2}{Q^2}\right)$$

- ❑ Factorization scale: μ_F^2

  ⟶ **To separate collinear from non-collinear contribution**

Recall: renormalization scale to separate local from non-local contribution

Calculation of perturbative parts

- **Use DIS structure function F_2 as an example:**

$$F_{2h}(x_B,Q^2)=\sum_{q,f}C_{q/f}\left(\frac{x_B}{x},\frac{Q^2}{\mu_F^2},\alpha_s\right)\otimes\varphi_{f/h}\left(x,\mu_F^2\right)+O\left(\frac{\Lambda_{\mathrm{QCD}}^2}{Q^2}\right)$$

- **Apply the factorized formula to parton states:** $h\to q$

Feynman diagrams → $F_{2q}(x_B,Q^2)=\sum_{q,f}C_{q/f}\left(\frac{x_B}{x},\frac{Q^2}{\mu_F^2},\alpha_s\right)\otimes\varphi_{f/q}\left(x,\mu_F^2\right)$ ← Feynman diagrams

- **Express both SFs and PDFs in terms of powers of α_s:**

0th order:

$$F_{2q}^{(0)}(x_B,Q^2)=C_q^{(0)}(x_B/x,Q^2/\mu_F^2)\otimes\varphi_{q/q}^{(0)}\left(x,\mu_F^2\right)$$

$$\Longrightarrow\quad \boxed{C_q^{(0)}(x)=F_{2q}^{(0)}(x)}\qquad \varphi_{q/q}^{(0)}(x)=\delta_{qq}\delta(1-x)$$

1th order:

$$F_{2q}^{(1)}(x_B,Q^2)=C_q^{(1)}(x_B/x,Q^2/\mu_F^2)\otimes\varphi_{q/q}^{(0)}\left(x,\mu_F^2\right)+C_q^{(0)}(x_B/x,Q^2/\mu_F^2)\otimes\varphi_{q/q}^{(1)}\left(x,\mu_F^2\right)$$

$$\Longrightarrow\quad \boxed{C_q^{(1)}(x,Q^2/\mu_F^2)=F_{2q}^{(1)}(x,Q^2)-F_{2q}^{(0)}(x,Q^2)\otimes\varphi_{q/q}^{(1)}\left(x,\mu_F^2\right)}$$

Leading order coefficient function

- **Projection operators for SFs:**

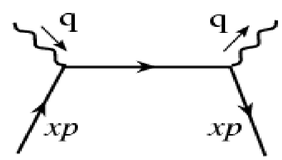
$$W_{\mu\nu} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) F_1\left(x,Q^2\right) + \frac{1}{p\cdot q}\left(p_\mu - q_\mu \frac{p\cdot q}{q^2}\right)\left(p_\nu - q_\nu \frac{p\cdot q}{q^2}\right) F_2\left(x,Q^2\right)$$

$$F_1(x,Q^2) = \frac{1}{2}\left(-g^{\mu\nu} + \frac{4x^2}{Q^2} p^\mu p^\nu\right) W_{\mu\nu}(x,Q^2)$$

$$F_2(x,Q^2) = x\left(-g^{\mu\nu} + \frac{12x^2}{Q^2} p^\mu p^\nu\right) W_{\mu\nu}(x,Q^2)$$

- **0th order:**

$$F_{2q}^{(0)}(x) = x g^{\mu\nu} W_{\mu\nu,q}^{(0)} = x g^{\mu\nu} \left[\frac{1}{4\pi}\ \cdots\ \right]$$



$$= \left(x g^{\mu\nu}\right)\frac{e_q^2}{4\pi} \mathrm{Tr}\left[\frac{1}{2}\gamma\cdot p\gamma_\mu \gamma\cdot\left(p+q\right)\gamma_\nu\right] 2\pi\delta\left((p+q)^2\right)$$

$$= e_q^2 x\delta(1-x)$$

$$C_q^{(0)}(x) = e_q^2 x\delta(1-x)$$

NLO coefficient function

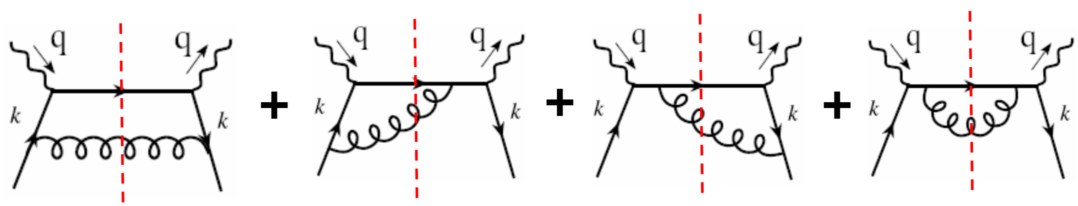
$$C_q^{(1)}(x,Q^2/\mu_F^2) = F_{2q}^{(1)}(x,Q^2) - F_{2q}^{(0)}(x,Q^2) \otimes \varphi_{q/q}^{(1)}\left(x,\mu_F^2\right)$$

- **Projection operators in n-dimension:** $g_{\mu\nu}g^{\mu\nu} = n \equiv 4 - 2\varepsilon$

$$(1-\varepsilon)F_2 = x\left(-g^{\mu\nu} + (3-2\varepsilon)\frac{4x^2}{Q^2}p^\mu p^\nu\right)W_{\mu\nu}$$

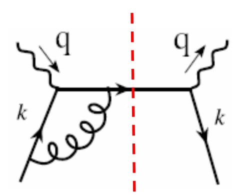
- **Feynman diagrams:**

$W_{\mu\nu,q}^{(1)}$

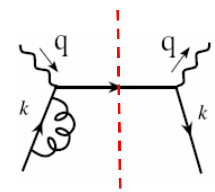


Real

Virtual { +



+ c.c. [+



+ c.c. + UV CT]

- **Calculation:** $-g^{\mu\nu}W_{\mu\nu,q}^{(1)}$ and $p^\mu p^\nu W_{\mu\nu,q}^{(1)}$

Contribution from the trace of Wμν

❑ Lowest order in n-dimension:

$$-g^{\mu\nu}W^{(0)}_{\mu\nu,q} = e_q^2(1-\varepsilon)\delta(1-x)$$

❑ NLO virtual contribution:

$$-g^{\mu\nu}W^{(1)V}_{\mu\nu,q} = e_q^2(1-\varepsilon)\delta(1-x)$$

$$*\left(-\frac{\alpha_s}{\pi}\right)C_F\left[\frac{4\pi\mu_F^2}{Q^2}\right]^{\varepsilon}\frac{\Gamma(1+\varepsilon)\Gamma^2(1-\varepsilon)}{\Gamma(1-2\varepsilon)}\left[\frac{1}{\varepsilon^2}+\frac{3}{2}\frac{1}{\varepsilon}+4\right]$$

❑ NLO real contribution:

$$-g^{\mu\nu}W^{(1)R}_{\mu\nu,q} = e_q^2(1-\varepsilon)C_F\left(-\frac{\alpha_s}{2\pi}\right)\left[\frac{4\pi\mu_F^2}{Q^2}\right]^{\varepsilon}\frac{\Gamma(1+\varepsilon)}{\Gamma(1-2\varepsilon)}$$

$$*\left\{-\frac{1-\varepsilon}{\varepsilon}\left[1-x+\left(\frac{2x}{1-x}\right)\left(\frac{1}{1-2\varepsilon}\right)\right]+\frac{1-\varepsilon}{2(1-2\varepsilon)(1-x)}+\frac{2\varepsilon}{1-2\varepsilon}\right\}$$

❑ The "+" distribution:

$$\left(\frac{1}{1-x}\right)^{1+\varepsilon} = -\frac{1}{\varepsilon}\delta(1-x) + \frac{1}{(1-x)_+} + \varepsilon\left(\frac{\ell n(1-x)}{1-x}\right)_+ + O\left(\varepsilon^2\right)$$

$$\int_z^1 dx \frac{f(x)}{(1-x)_+} \equiv \int_z^1 dx \frac{f(x)-f(1)}{1-x} + \ell n(1-z)\, f(1)$$

❑ One loop contribution to the trace of Wμν:

$$-g^{\mu\nu}W^{(1)}_{\mu\nu,q} = e_q^2(1-\varepsilon)\left(\frac{\alpha_s}{2\pi}\right)\left\{-\frac{1}{\varepsilon}P_{qq}(x) + P_{qq}(x)\ell n\left(\frac{Q^2}{\mu_F^2(4\pi e^{-\gamma_E})}\right)\right.$$

$$+C_F\left[\left(1+x^2\right)\left(\frac{\ell n(1-x)}{1-x}\right)_+ - \frac{3}{2}\left(\frac{1}{1-x}\right)_+ - \frac{1+x^2}{1-x}\ell n(x)\right.$$

$$\left.\left. + 3 - x - \left(\frac{9}{2}+\frac{\pi^2}{3}\right)\delta(1-x)\right]\right\}$$

❑ Splitting function:

$$P_{qq}(x) = C_F\left[\frac{1+x^2}{(1-x)_+} + \frac{3}{2}\delta(1-x)\right]$$

❑ One loop contribution to $p^\mu p^\nu W_{\mu\nu}$:

$$p^\mu p^\nu W^{(1)V}_{\mu\nu,q} = 0 \qquad\qquad p^\mu p^\nu W^{(1)R}_{\mu\nu,q} = e_q^2 C_F \frac{\alpha_s}{2\pi}\frac{Q^2}{4x}$$

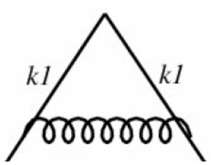
❑ One loop contribution to F_2 of a quark:

$$F^{(1)}_{2q}(x,Q^2) = e_q^2 x \frac{\alpha_s}{2\pi}\left\{\left(-\frac{1}{\varepsilon}\right)_{\rm CO} P_{qq}(x)\left(1+\varepsilon\ell n(4\pi e^{-\gamma_E})\right) + P_{qq}(x)\ell n\left(\frac{Q^2}{\mu_F^2}\right)\right.$$
$$\left. + C_F\left[(1+x^2)\left(\frac{\ell n(1-x)}{1-x}\right)_+ - \frac{3}{2}\left(\frac{1}{1-x}\right)_+ - \frac{1+x^2}{1-x}\ell n(x) + 3 + 2x - \left(\frac{9}{2}+\frac{\pi^2}{3}\right)\delta(1-x)\right]\right\}$$

$$\Rightarrow \quad \infty \quad \text{as} \;\; \varepsilon \to 0$$

❑ One loop contribution to quark PDF of a quark:

$$\varphi^{(1)}_{q/q}(x,\mu_F^2) = \left(\frac{\alpha_s}{2\pi}\right) P_{qq}(x)\left\{\left(\frac{1}{\varepsilon}\right)_{\rm UV} + \left(-\frac{1}{\varepsilon}\right)_{\rm CO}\right\} + \text{UV-CT}$$



Different UV-CT = different factorization scheme!

❑ Common UV-CT terms:

- ❖ **MS scheme:** $\text{UV-CT}\Big|_{\text{MS}} = -\frac{\alpha_s}{2\pi} P_{qq}(x)\left(\frac{1}{\varepsilon}\right)_{\text{UV}}$
- ❖ **$\overline{\text{MS}}$ scheme:** $\text{UV-CT}\Big|_{\overline{\text{MS}}} = -\frac{\alpha_s}{2\pi} P_{qq}(x)\left(\frac{1}{\varepsilon}\right)_{\text{UV}} \left(1+\varepsilon \ell n(4\pi e^{-\gamma_E})\right)$
- ❖ **DIS scheme:** **choose a UV-CT, such that** $C_q^{(1)}(x, Q^2/\mu_F^2)\Big|_{\text{DIS}} = 0$

❑ One loop coefficient function:

$$\boxed{C_q^{(1)}(x,Q^2/\mu_F^2) = F_{2q}^{(1)}(x,Q^2) - F_{2q}^{(0)}(x,Q^2) \otimes \varphi_{q/q}^{(1)}\left(x,\mu_F^2\right)}$$

$$C_q^{(1)}(x,Q^2/\mu^2) = e_q^2 x \frac{\alpha_s}{2\pi}\left\{ P_{qq}(x)\ell n\left(\frac{Q^2}{\mu_{\overline{\text{MS}}}^2}\right) + C_F\left[(1+x^2)\left(\frac{\ell n(1-x)}{1-x}\right)_+ - \frac{3}{2}\left(\frac{1}{1-x}\right)_+ - \frac{1+x^2}{1-x}\ell n(x) + 3 + 2x - \left(\frac{9}{2}+\frac{\pi^2}{3}\right)\delta(1-x)\right]\right\}$$

Dependence on factorization scale

- Physical cross sections should not depend on the factorization scale

$$\mu_F^2 \frac{d}{d\mu_F^2} F_2(x_B, Q^2) = 0$$

→ Evolution (differential-integral) equation for PDFs

$$\sum_f \left[\mu_F^2 \frac{d}{d\mu_F^2} C_f\left(\frac{x_B}{x}, \frac{Q^2}{\mu_F^2}, \alpha_s\right)\right] \otimes \varphi_f\left(x, \mu_F^2\right) + \sum_f C_f\left(\frac{x_B}{x}, \frac{Q^2}{\mu_F^2}, \alpha_s\right) \otimes \mu_F^2 \frac{d}{d\mu_F^2} \varphi_f\left(x, \mu_F^2\right) = 0$$

- PDFs and coefficient functions share the same logarithms

PDFs: $\log\left(\mu_F^2/\mu_0^2\right)$ or $\log\left(\mu_F^2/\Lambda_{\rm QCD}^2\right)$

Coefficient functions: $\log\left(Q^2/\mu_F^2\right)$ or $\log\left(Q^2/\mu^2\right)$

→ DGLAP evolution equation:

$$\mu_F^2 \frac{\partial}{\partial \mu_F^2} \varphi_i(x, \mu_F^2) = \sum_j P_{i/j}\left(\frac{x}{x'}, \alpha_s\right) \otimes \varphi_j(x', \mu_F^2)$$

DGLAP evolution of PDFs

❑ DGLAP equations:

$$\mu_F^2 \frac{\partial}{\partial \mu_F^2} \varphi_i(x, \mu_F^2) = \sum_j P_{i/j}\left(\frac{x}{x'}, \alpha_s\right) \otimes \varphi_j(x', \mu_F^2)$$

❑ Splitting functions:

- Splitting functions have to be process independent
- Can be then derived in many different ways
  - from the log part of the C's
  - from the anomalous dimension of the nonlocal operators defining the PDFs

❑ Predictive power of pQCD:

Once the boundary condition is fixed by the data, the scale dependence of PDFs is a prediction of pQCD

Global QCD analysis of PDFs

❑ PDFs are extracted by using:

❖ **DGLAP**
$$\mu_F^2 \frac{\partial}{\partial \mu_F^2} \varphi_i(x, \mu_F^2) = \sum_j P_{i/j}\left(\frac{x}{x'}, \alpha_s\right) \otimes \varphi_j(x', \mu_F^2)$$

❖ **Factorized hard cross sections, e.g.**
$$F_{2h}(x_B, Q^2) = \sum_q C_{q/f}\left(\frac{x_B}{x}, \frac{Q^2}{\mu_F^2}, \alpha_s\right) \otimes \varphi_{f/h}\left(x, \mu_F^2\right) + O\left(\frac{\Lambda_{\text{QCD}}^2}{Q^2}\right)$$

❖ **Data:** to fix the boundary condition of DGLAP

❑ The order and scheme dependence of PDFs:

❖ **Leading order (tree-level) C_q**

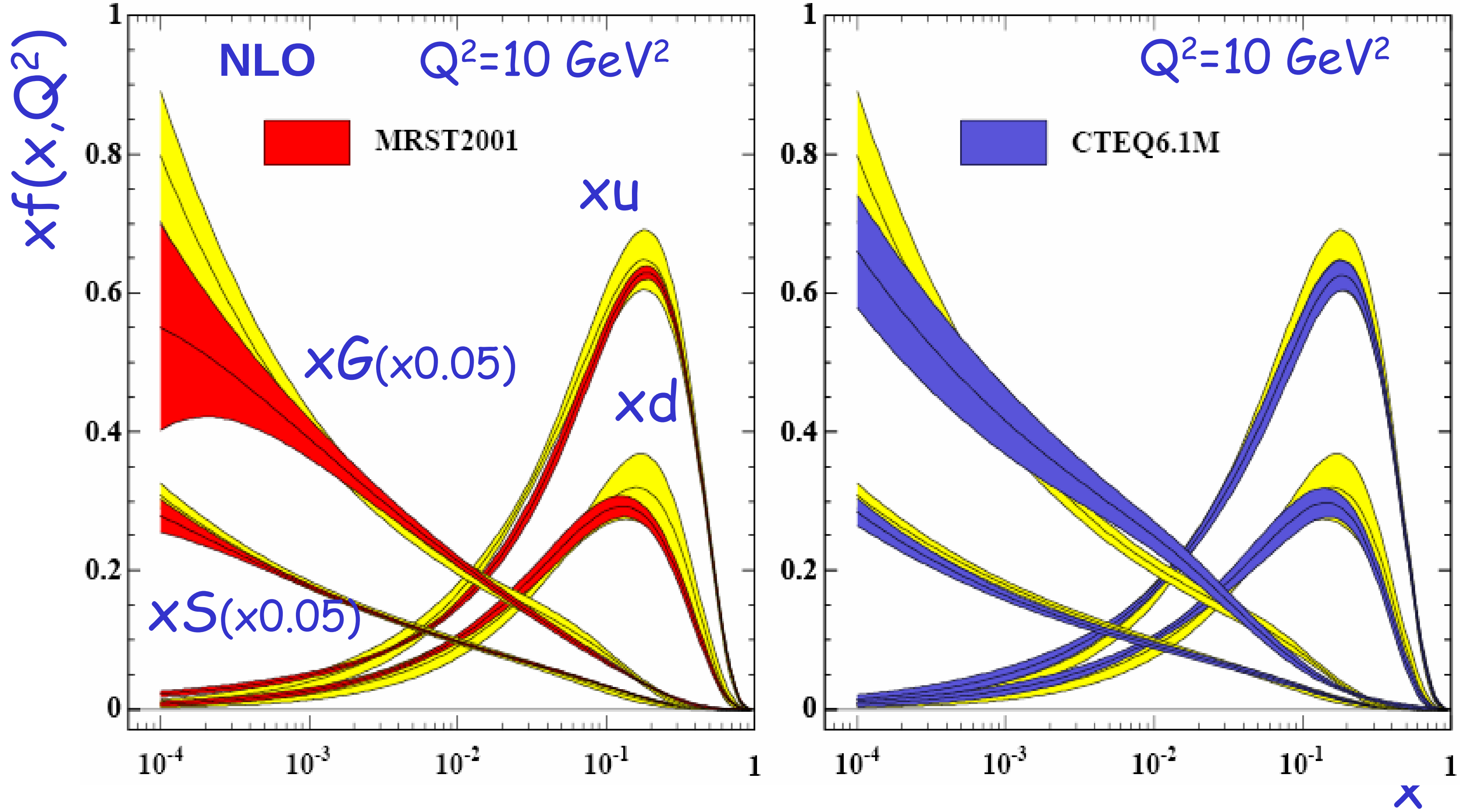
❖ **Next-to-Leading order C_q**

} ⟺ { **LO PDF's** / **NLO PDF's**

❖ **Calculation of C_q at NLO and beyond depends on the UVCT** → **the scheme dependence of C_q** → **the scheme dependence of PDFs**

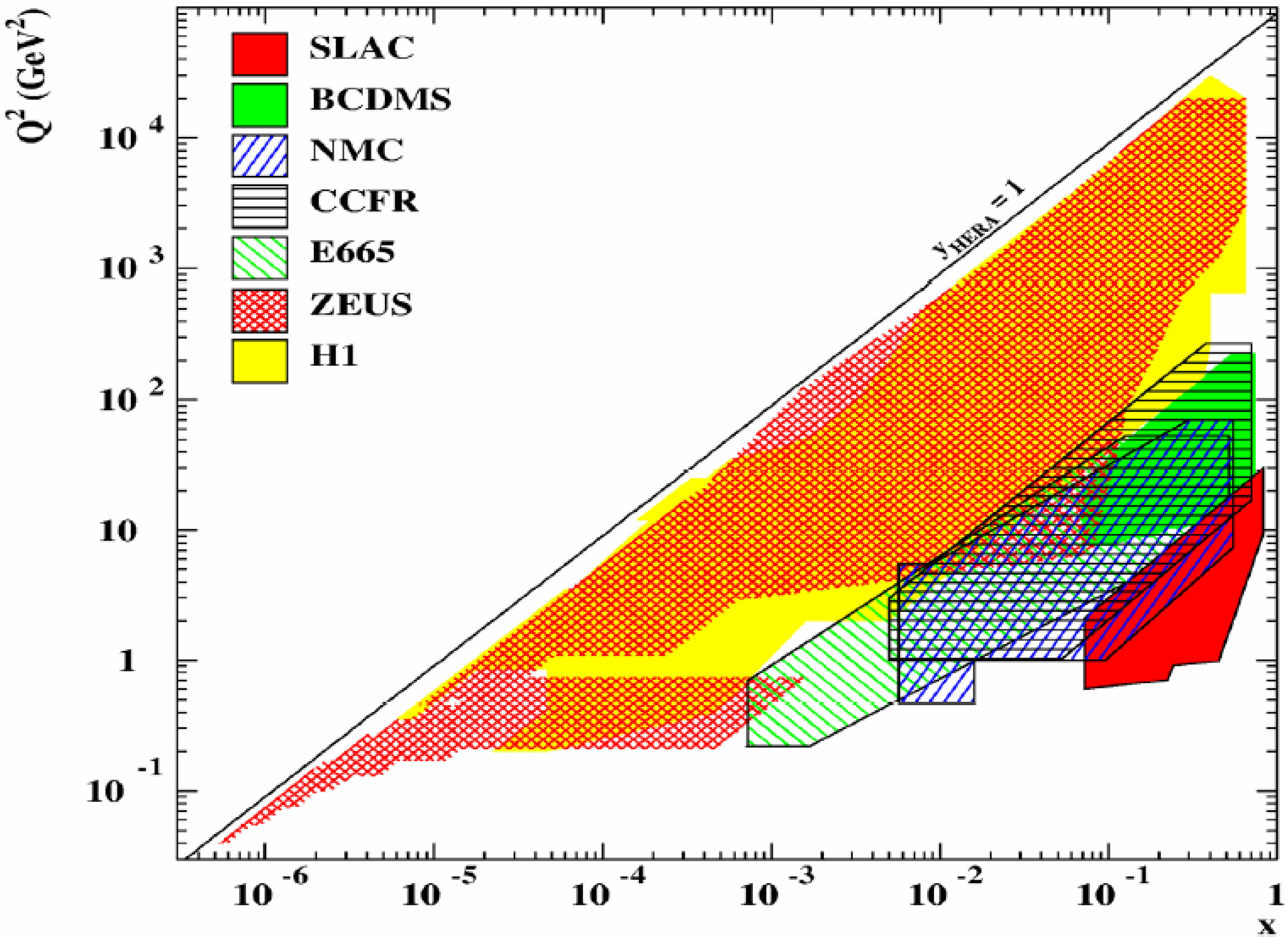
PDFs of a spin-averaged proton

❖ **Modern sets of PDFs with uncertainties:**

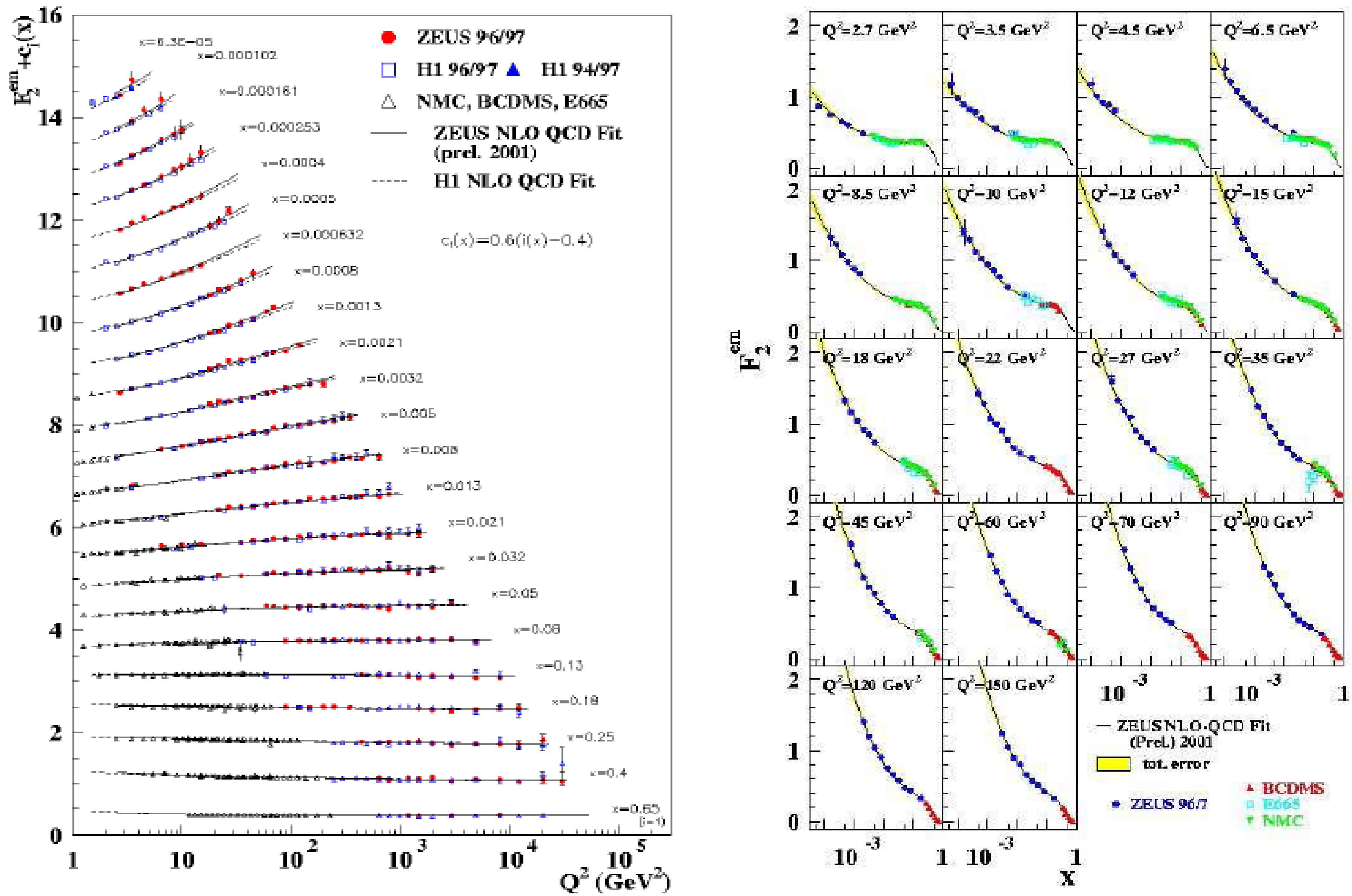


Consistently fit almost all data with Q > 2GeV

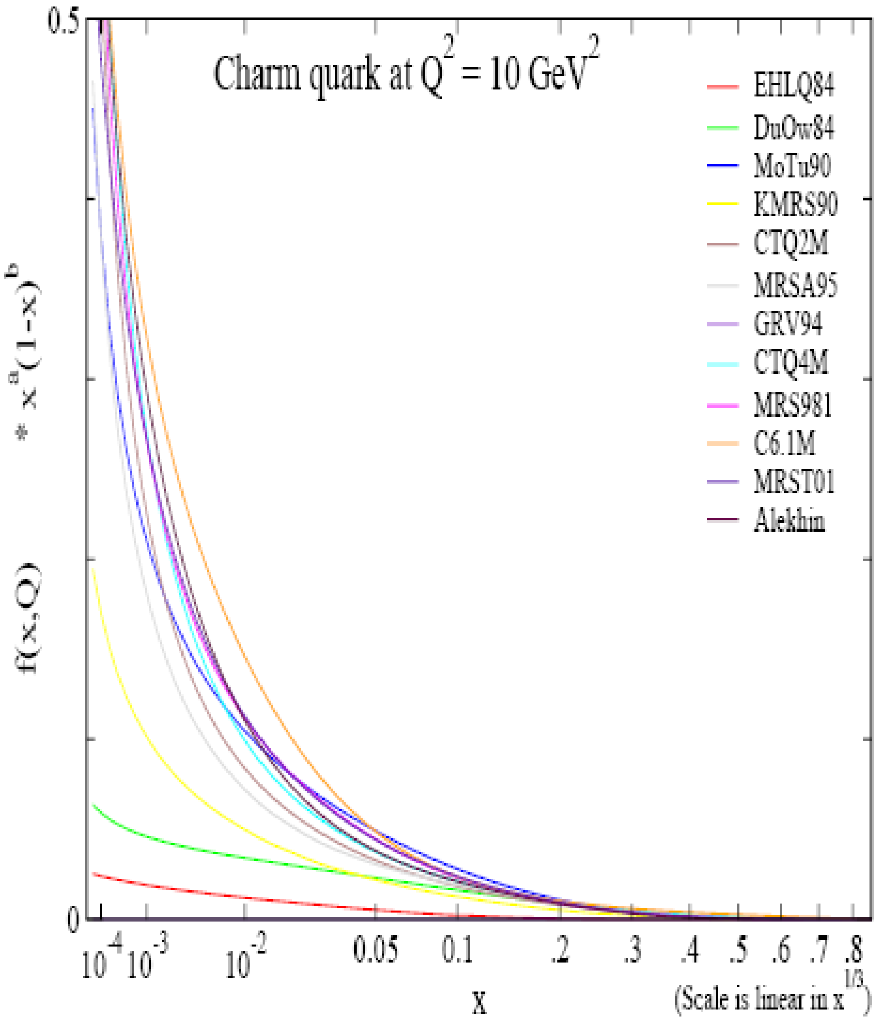
Kinematic Regions of DIS

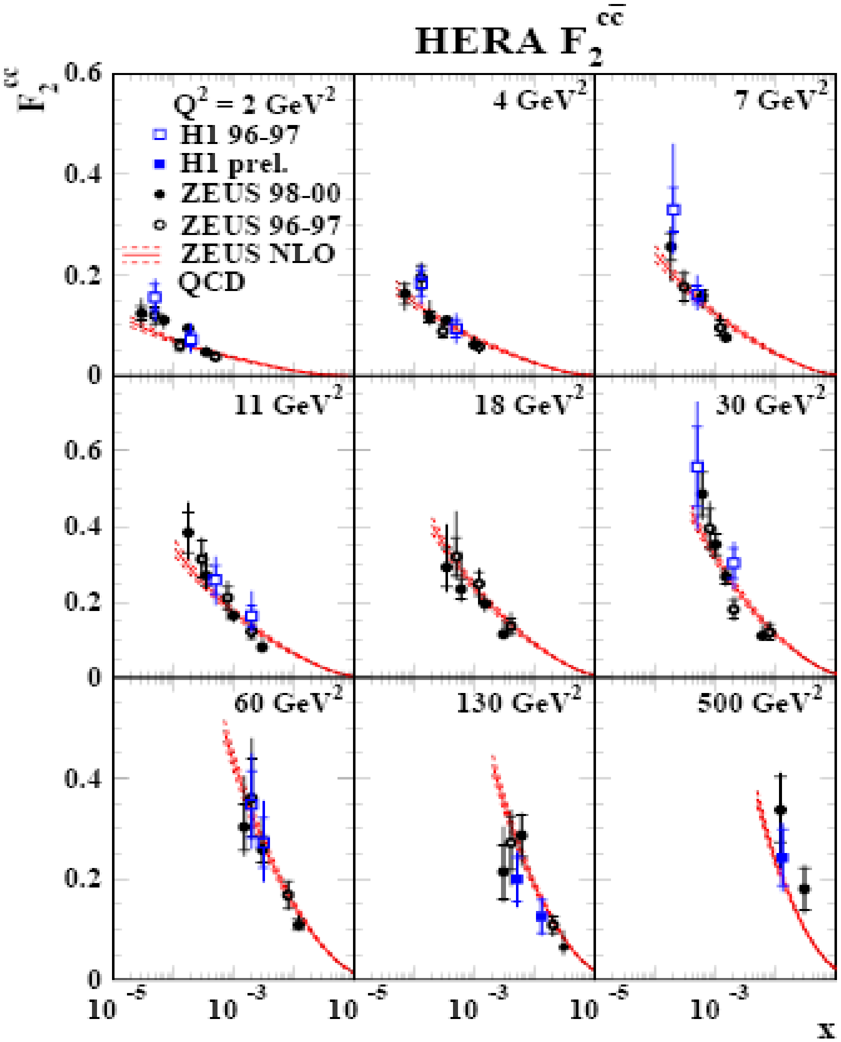


Comparison with DIS data



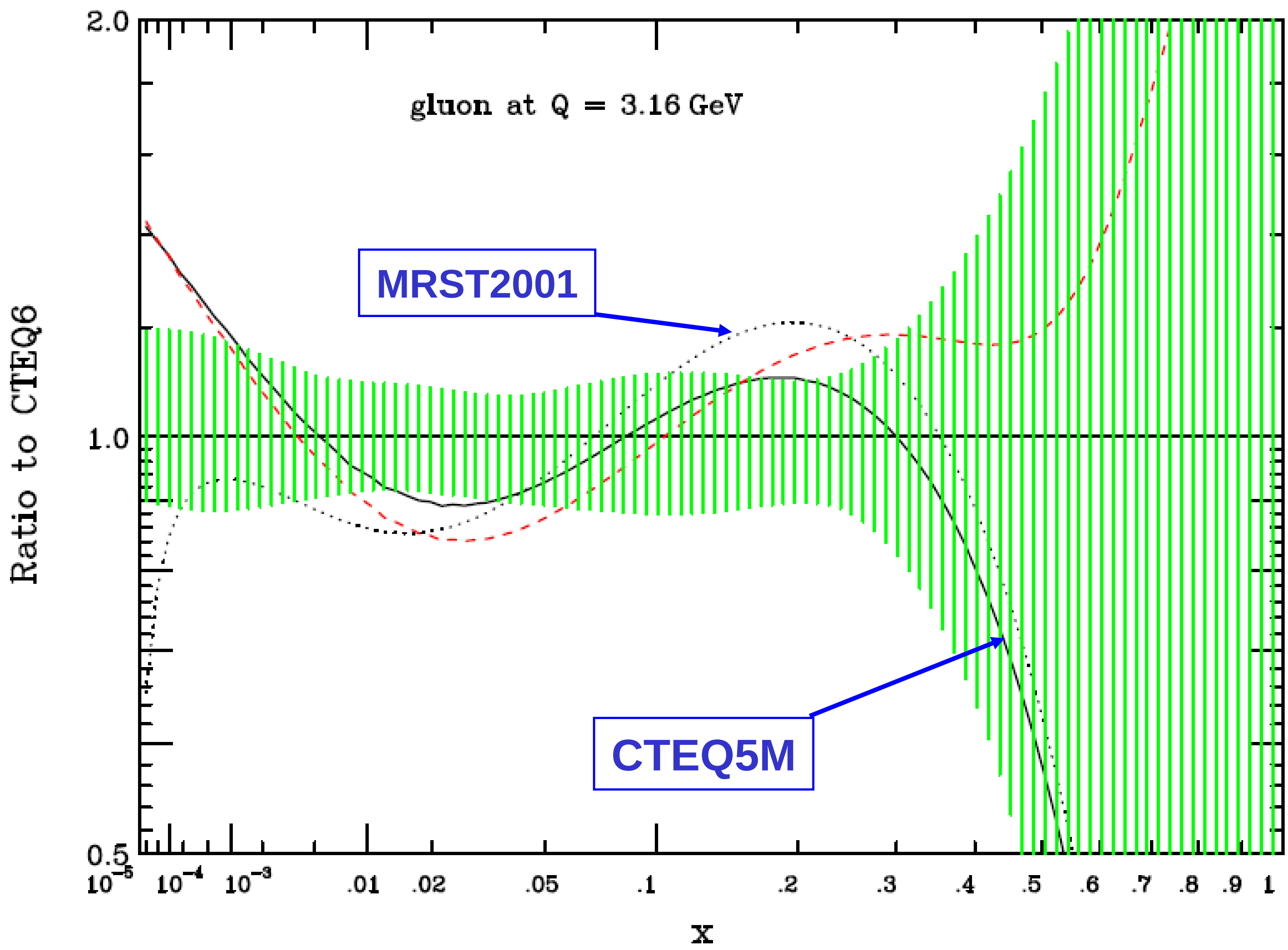
Charm quark distributions





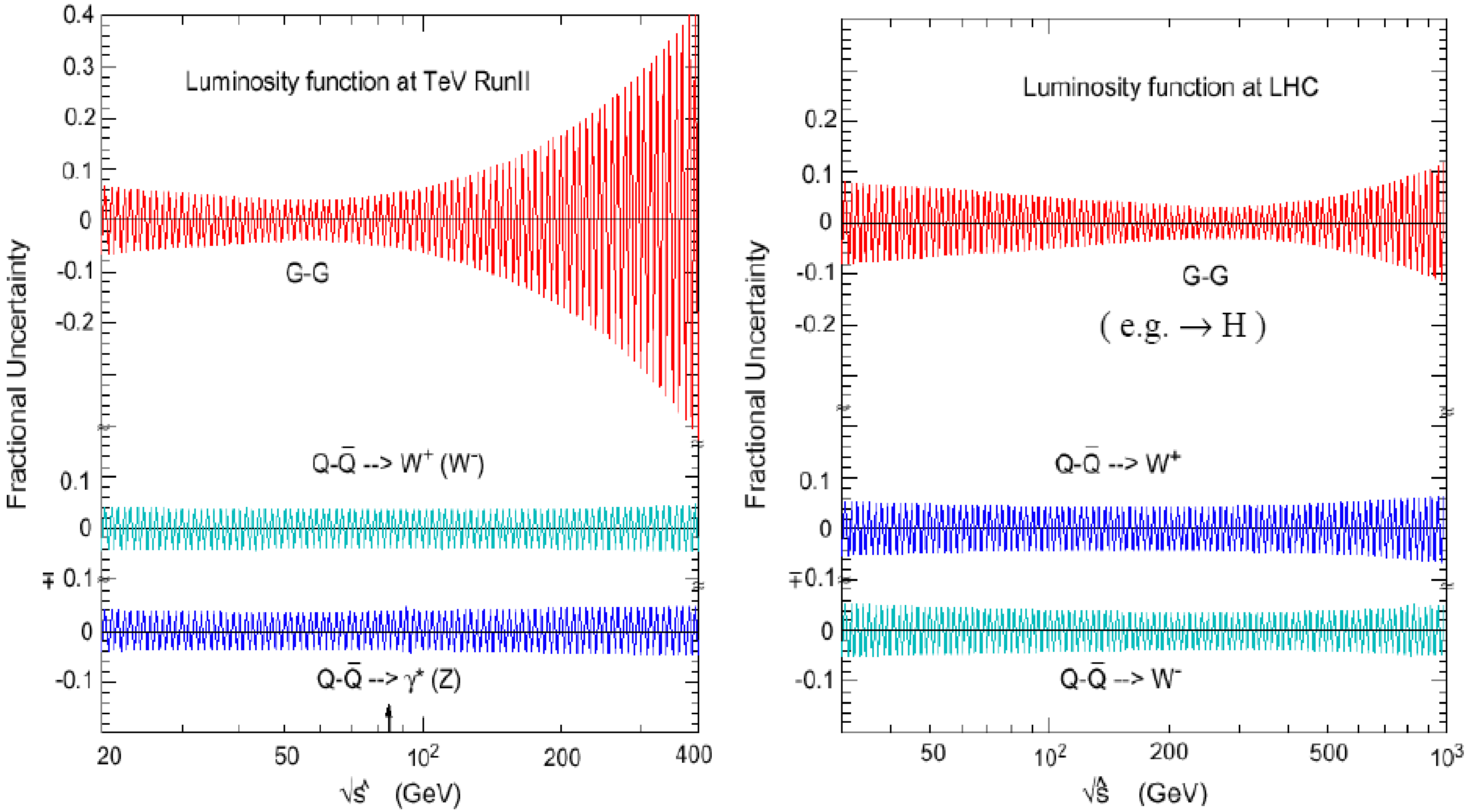
Large gluon at small-x ➡ large charm quark distribution

Uncertainties of gluon distribution



PDF uncertainty for observables

CTEQ6 PDFs



% uncertainty for strong interaction

Recover the effect of non-vanishing k_T

❑ Sources of power corrections:

- ❖ **Parton transverse momentum:** $\langle k_\perp^2 \rangle / Q^2 \sim \langle k^2 \rangle / Q^2$
- ❖ **Target and parton masses:** m^2 / Q^2
- ❖ **Coherent multiple scattering:** $\left[\left(1/Q^2 \right) / R^2 \right] \langle F^+_{\perp} F^{+\perp} \rangle \langle \text{Medium length} \rangle$

❑ Systematics of power corrections:

$$\sigma^h_{phys} = \hat{\sigma}^i_2 \otimes [1 + \alpha_s + \alpha_s^2 + \ldots] \otimes T_2^{i/h}(x) + \frac{\hat{\sigma}^i_4}{Q^2} \otimes [1 + \alpha_s + \alpha_s^2 + \ldots] \otimes T_4^{i/h}(x) + \frac{\hat{\sigma}^i_6}{Q^4} \otimes [1 + \alpha_s + \alpha_s^2 + \ldots] \otimes T_6^{i/h}(x) + \ldots$$

Leading Twist

perturbative

Power corrections

Factorization may not be true for power corrections!

Need to be proved for any given process

Qiu and Vitev, PRL 2004

Improvement from the fixed order

- Beyond the Born term (lowest order), partonic hard-parts are **NOT** unique, due to renormalization of parton distributions

- Once $\varphi(x,\mu^2)$ is fixed in one scheme, same $\varphi(x,\mu^2)$ should be used for all calculations of partonic parts

- Coefficient has the $P_{qq}(x)\ell n\left(\frac{Q^2}{\mu_F^2}\right)$

  Suggests to choose the scale: $\mu_F^2 \sim Q^2$

- Coefficient has potentially large logarithms:

$$\ell n(x), \quad \frac{1}{(1-x)_+}, \quad \left(\frac{\ell n(1-x)}{1-x}\right)_+$$

$\Longrightarrow$ Resummation of the large logarithms

Summary

- We can actually "see" and "count" the quarks and gluons – quark and gluon distributions

- PQCD factorization works for DIS to all orders as well as all powers due to OPE

- PDFs evolves – number of partons is sensitive to the probing scale

- PQCD global analysis for spin averaged cross sections results into the reasonably well-determined universal PDFs

What happen if there are more than one identified hadrons?