Introduction to Perturbative QCD

Lecture 2

Jianwei Qiu
Iowa State University/Argonne National Laboratory

PHENIX Spinfest at RIKEN 2007
June 11 - July 27, 2007
RIKEN Wako Campus, Wako, Japan

Outline for Lecture 2

- ❑ Infrared Safety
- ❑ Purely infrared safe cross sections
- ❑ Jets – "trace" of the partons
- ❑ General conditions for purely infrared safe observables

Excellent resource – CTEQ summer school website

http://www.phys.psu.edu/~cteq

Infrared Safety

❑ **Running quark mass:**

$$m(\mu_2) = m(\mu_1)\, \exp\left[-\int_{\mu_1}^{\mu_2} \frac{d\lambda}{\lambda}\left[1+\gamma_m(g(\lambda))\right] \right] \;\Rightarrow\; 0 \;\text{ as } \mu_2 \to \infty$$

Perturbation theory becomes a massless theory when $\mu \to \infty$

❑ **Infrared safety:**

$$\sigma_{\text{phy}}\left(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2), \frac{m^2(\mu^2)}{\mu^2}\right) \Rightarrow \hat{\sigma}\left(\frac{Q^2}{\mu^2}, \alpha_s(\mu^2)\right) + O\left[\left(\frac{m^2(\mu^2)}{\mu^2}\right)^{\kappa}\right]$$

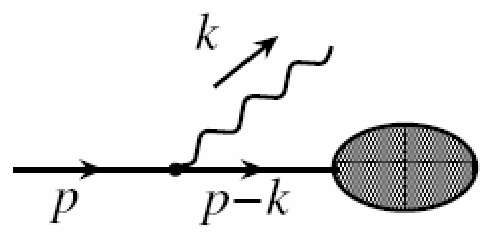
Infrared safe = $\kappa > 0$

Asymptotic freedom is useful for quantities that are infrared safe

QCD perturbation theory ($Q \gg \Lambda_{QCD}$) is effectively a massless theory

Infrared and collinear divergence

Consider a general diagram:



$p^2 = 0, \quad k^2 = 0$ for a massless theory

singularity

❖ $k^\mu \to 0 \;\Rightarrow\; (p-k)^2 \to p^2 = 0$

⟹ **Infrared (IR) divergence**

❖ $k^\mu \parallel p^\mu \;\Rightarrow\; k = \lambda p$ with $0 < \lambda < 1$

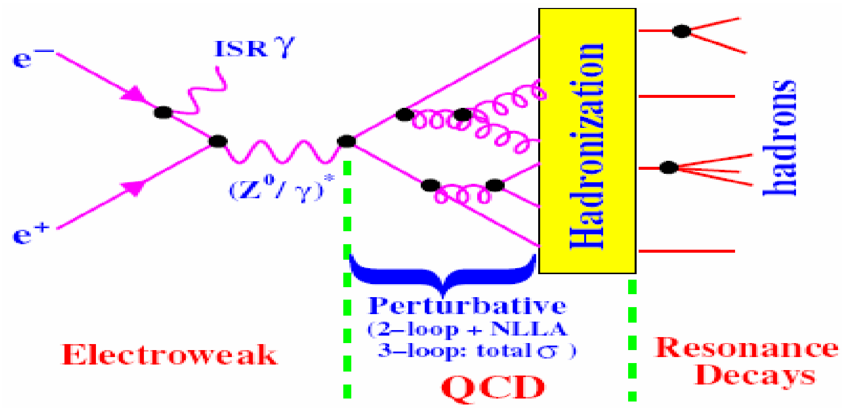
$$\Rightarrow (p-k)^2 \to (1-\lambda)^2 p^2 = 0$$

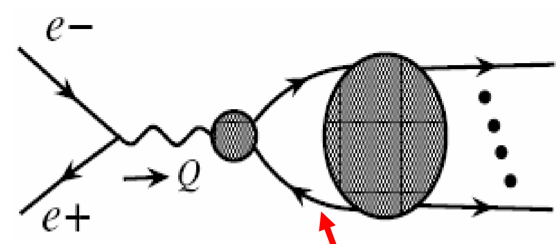
⟹ **Collinear (CO) divergence**

IR and CO divergences are generic problems for massless perturbation theory

Purely Infrared safe cross sections

e+e- → hadron total cross section is infrared safe (IRS)





Hadrons "n"

Partons "m"

If there is no quantum interference between partons and hadrons,

$$\sigma^{\text{tot}}_{e^+e^-\to\text{hadrons}} \propto \sum_n \mathrm{P}_{e^+e^-\to n} = \sum_n \sum_m \mathrm{P}_{e^+e^-\to m}\mathrm{P}_{m\to n} = \sum_m \mathrm{P}_{e^+e^-\to m} \sum_n \mathrm{P}_{m\to n} = 1$$

$$\sigma^{\text{tot}}_{e^+e^-\to\text{partons}} \propto \sum_m \mathrm{P}_{e^+e^-\to m}$$

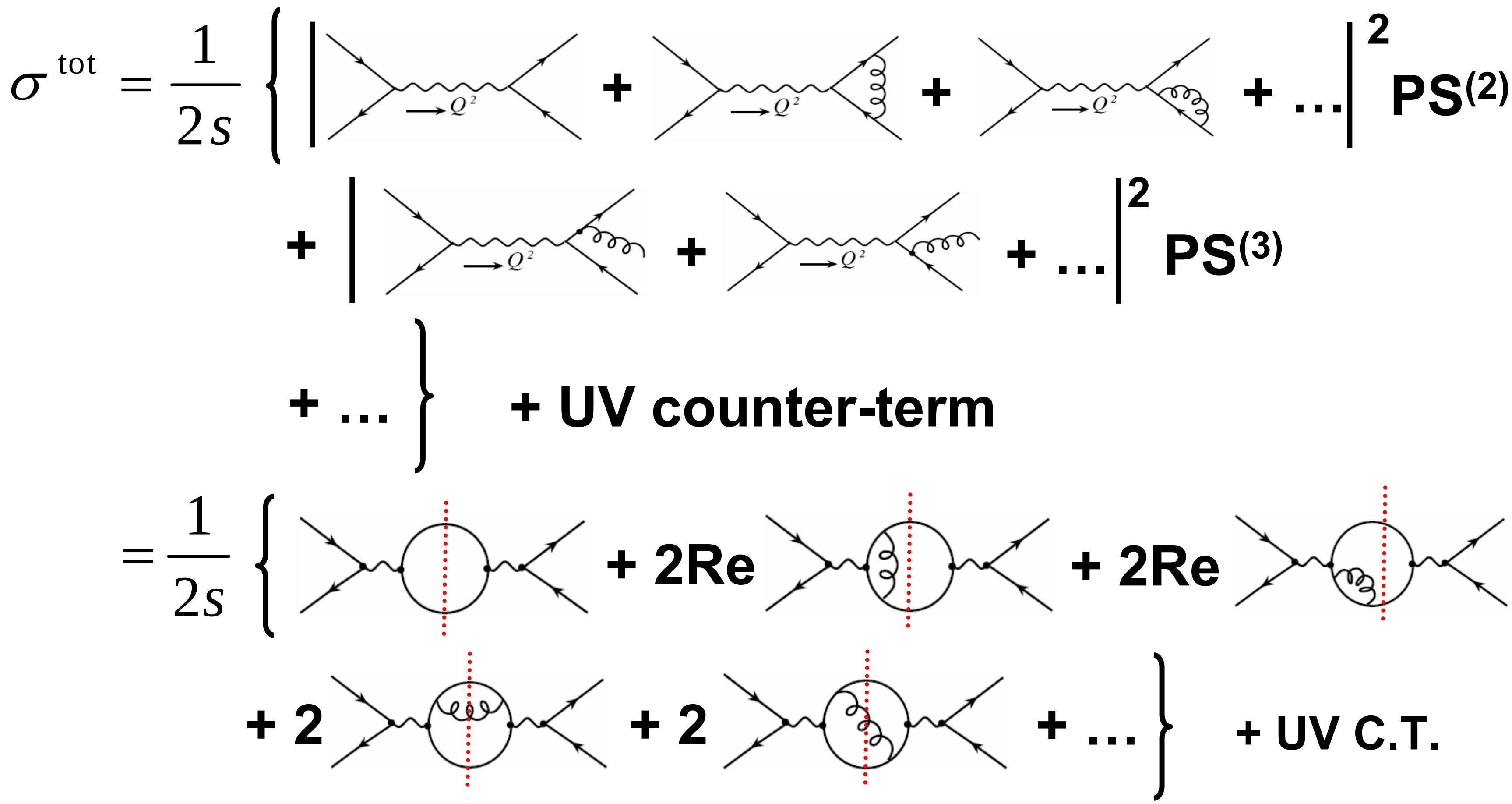
Unitarity

$$\Rightarrow \quad \sigma^{\text{tot}}_{e^+e^-\to\text{hadrons}} = \sigma^{\text{tot}}_{e^+e^-\to\text{partons}}$$

Finite in perturbation Theory – KLN theorem

"Local" – of order of 1/Q

σ^{tot} for $e^+e^- \rightarrow$ hadrons in pQCD



$$= \sigma_2^{(0)} + \sigma_2^{(1)} + \sigma_3^{(1)}$$

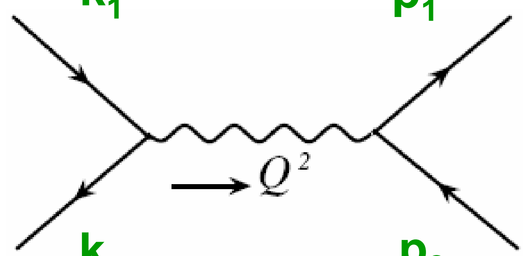
Born

$O(\alpha_s)$

3-particle phase space

Leading order contribution – I

- **Lowest order Feynman diagram:**



$$s = (k_1 + k_2)^2$$
$$t = (k_1 - p_1)^2$$
$$u = (k_2 - p_1)^2$$

- **Invariant amplitude square:**

$$|\bar{M}_{e^+e^- \to Q\bar{Q}}|^2 = e^4 e_Q^2 N_c \frac{1}{s^2} \frac{1}{2^2} \mathrm{Tr}\left[\gamma \cdot k_2 \gamma^\mu \gamma \cdot k_1 \gamma^\nu\right]$$
$$\times \mathrm{Tr}\left[\left(\gamma \cdot p_1 + m_Q\right)\gamma_\mu \left(\gamma \cdot p_2 - m_Q\right)\gamma_\nu\right]$$
$$= e^4 e_Q^2 N_c \frac{2}{s^2}\left[(m_Q^2 - t)^2 + (m_Q^2 - u)^2 + 2m_Q^2 s\right]$$

Keeps the final state quark mass

Leading order contribution – II

- **Lowest order total cross section:**

$$\frac{d\sigma_{e^+e^-\to Q\bar{Q}}}{dt} = \frac{1}{16\pi s^2}\,|\,\bar{M}_{e^+e^-\to Q\bar{Q}}\,|^2 \qquad \text{where} \quad s = Q^2$$

Threshold constraint

$$\sigma_2^{(0)} = \sum_Q \sigma_{e^+e^-\to Q\bar{Q}} = \sum_Q e_Q^2 N_c \frac{4\pi\alpha_{em}^2}{3s}\left[1+\frac{2m_Q^2}{s}\right]\sqrt{1-\frac{4m_Q^2}{s}}$$

One of the best tests for the number of colors

- **Normalized total cross section:**

$$R = \frac{\sigma_{e^+e^-\to \text{hadrons}}}{\sigma_{e^+e^-\to\mu^+\mu^-}} = \frac{\sum_Q \sigma_{e^+e^-\to Q\bar{Q}}}{\sigma_{e^+e^-\to\mu^+\mu^-}} = \sum_Q e_Q^2\, N_c \left[1+\frac{2m_Q^2}{s}\right]\sqrt{1-\frac{4m_Q^2}{s}}$$

One of the best measurements for the N_c

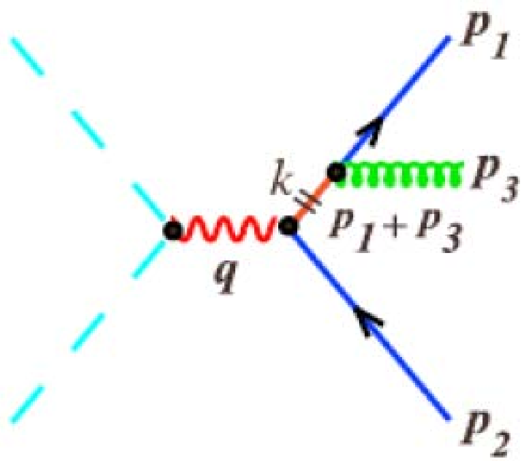
Next-to-Leading order contribution – I

- **Real Feynman diagram:**

$$x_i = \frac{E_i}{\sqrt{s}/2} = \frac{2\,p_i.q}{s} \qquad \text{with } i = 1, 2, 3$$

$$\sum_i x_i = \frac{2\left(\sum_i p_i\right).q}{s} = 2$$

$$2\left(1 - x_1\right) = x_2 x_3 \left(1 - \cos\theta_{23}\right), \quad cycl.$$



+ crossing

- **Contribution to the cross section:**

$$\frac{1}{\sigma_0}\frac{d\sigma_{e^+e^- \to Q\bar{Q}g}}{dx_1 dx_2} = \frac{\alpha_s}{2\pi} C_F \frac{x_1^2 + x_2^2}{\left(1 - x_1\right)\left(1 - x_2\right)}$$

IR as x3→0
CO as $\theta_{13} \to 0$
$\theta_{23} \to 0$

Divergent as $x_i \to 1$
Need the virtual contribution and a regulator!

Next-to-Leading order contribution – II

- **Infrared regulator:**
  - Gluon mass: $m_g \neq 0$
    - easier because all integrals at one-loop is finite
  - Dimensional regularization: $4 \rightarrow D = 4 - 2\varepsilon$
    – menifestly preserves gauge invariance

- **Gluon mass regulator:**

$$\sigma^{(1)}_{3,m_g} = \sigma^{(0)}_2 \frac{4}{3}\left(\frac{\alpha_s}{\pi}\right)\left[2\ln^2\left(\frac{Q}{m_g}\right) - 3\ln\left(\frac{Q}{m_g}\right) - \frac{\pi^2}{6} - \frac{5}{2}\right]$$

$$\sigma^{(1)}_{2,m_g} = \sigma^{(0)}_2 \frac{4}{3}\left(\frac{\alpha_s}{\pi}\right)\left[-2\ln^2\left(\frac{Q}{m_g}\right) + 3\ln\left(\frac{Q}{m_g}\right) + \frac{\pi^2}{6} - \frac{7}{4}\right]$$

$$\sigma^{\text{tot}} = \sigma^{(0)}_2 + \sigma^{(1)}_{3,m_g} + \sigma^{(1)}_{2,m_g} + O\left(\alpha_s^2\right) = \sigma^{(0)}_2\left[1 + \frac{\alpha_s}{\pi}\right] + O\left(\alpha_s^2\right)$$

No m_g dependence!

Next-to-Leading order contribution – III

❑ Dimensional regulator:

$$\sigma_{3,\varepsilon}^{(1)} = \sigma_{2,\varepsilon}^{(0)} \frac{4}{3}\left(\frac{\alpha_s}{\pi}\right)\left(\frac{4\pi\mu^2}{Q^2}\right)^{\varepsilon}\left[\frac{\Gamma(1-\varepsilon)^2}{\Gamma(1-3\varepsilon)}\right]\left[\frac{1}{\varepsilon^2}+\frac{3}{2\varepsilon}+\frac{19}{4}\right]$$

$$\sigma_{2,\varepsilon}^{(1)} = \sigma_{2,\varepsilon}^{(0)} \frac{4}{3}\left(\frac{\alpha_s}{\pi}\right)\left(\frac{4\pi\mu^2}{Q^2}\right)^{\varepsilon}\left[\frac{\Gamma(1-\varepsilon)^2\,\Gamma(1+\varepsilon)}{\Gamma(1-2\varepsilon)}\right]\left[-\frac{1}{\varepsilon^2}-\frac{3}{2\varepsilon}+\frac{\pi^2}{2}-4\right]$$

$$\sigma_{3,\varepsilon}^{(1)} + \sigma_{2,\varepsilon}^{(1)} = \sigma_{2}^{(0)}\left[\frac{\alpha_s}{\pi} + O(\varepsilon)\right]$$

No ε dependence!

$$\sigma^{\text{tot}} = \sigma_2^{(0)} + \sigma_{3,\varepsilon}^{(1)} + \sigma_{2,\varepsilon}^{(1)} + O\left(\alpha_s^2\right) = \sigma_2^{(0)}\left[1+\frac{\alpha_s}{\pi}\right] + O\left(\alpha_s^2\right)$$

❑ Lesson:

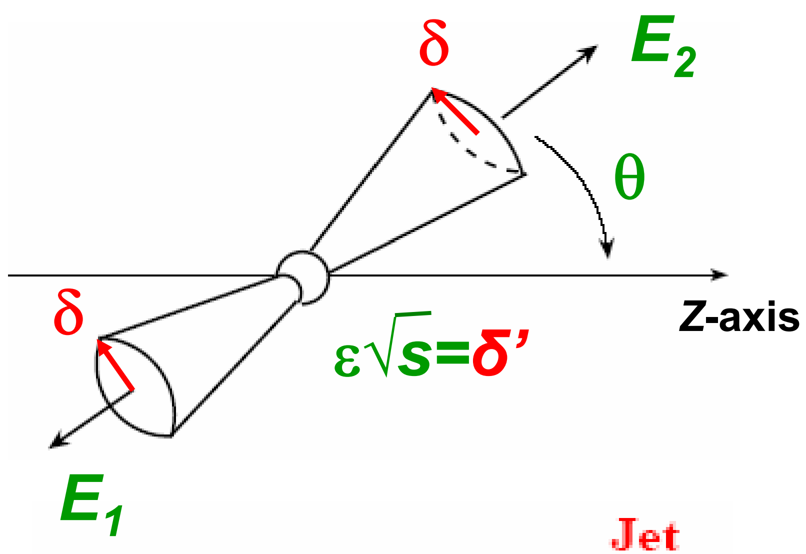
σ^{tot} is independent of the choice of IR and CO regularization

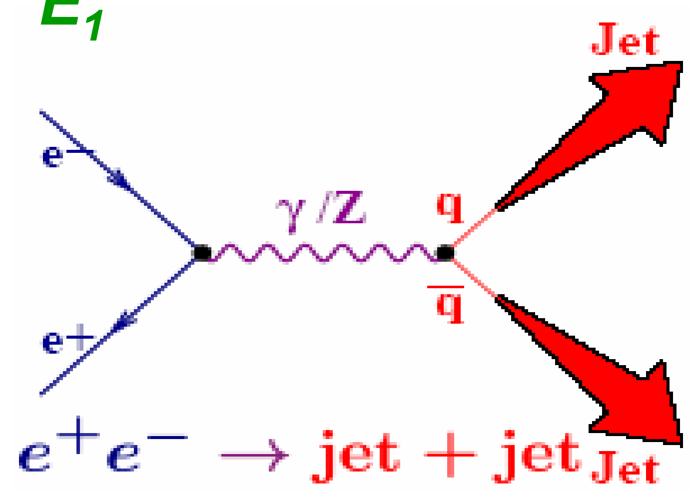
⟹ **σ^{tot} is Infrared safe!**

Jets in e^+e^- - trace of the partons

- ❑ **Jets – Inclusive x-section with a limited phase-space**
- ❑ **Q: will IR cancellation be completed?**
  - ❖ **Leading partons are moving away from each other**
  - ❖ **Soft gluon interactions should not change the direction of an energetic parton → a "jet" – "trace" of a parton**
  - ❖ **Jet algorithm**

Sterman-Weinberg Jet

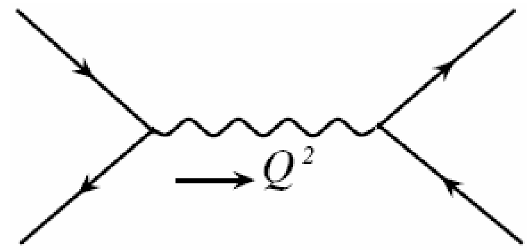




Two-jet cross section in e^+e^-

❑ **Parton-Model = Born term in QCD:**

$$\sigma_{2\text{Jet}}^{(\text{PM})} = \frac{3}{8}\sigma_0\left(1+\cos^2\theta\right)$$



❑ **Two-jet in pQCD:**

$$\sigma_{2\text{Jet}}^{(\text{pQCD})} = \frac{3}{8}\sigma_0\left(1+\cos^2\theta\right)\left(1+\sum_{n=1} C_n\left(\frac{\alpha_s}{\pi}\right)^n\right)$$

with $\quad C_n = C_n(\delta)$

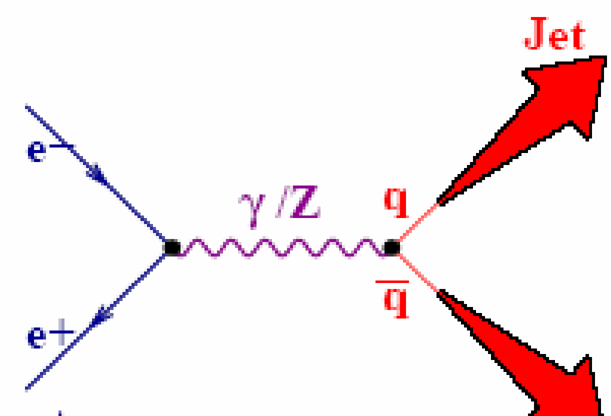
❑ **Sterman-Weinberg jet:**

$$\sigma_{2\text{Jet}}^{(\text{pQCD})} = \frac{3}{8}\sigma_0\left(1+\cos^2\theta\right)\left[1-\frac{4}{3}\frac{\alpha_s}{\pi}\left(4\ln(\delta)\ln(\delta') + 3\ln(\delta) + \frac{\pi^2}{3} + \frac{5}{2}\right)\right]$$

$$\sigma_{\text{total}} = \sigma_{2\text{Jet}} \qquad \text{as} \quad Q\to\infty$$

A clean two-jet event

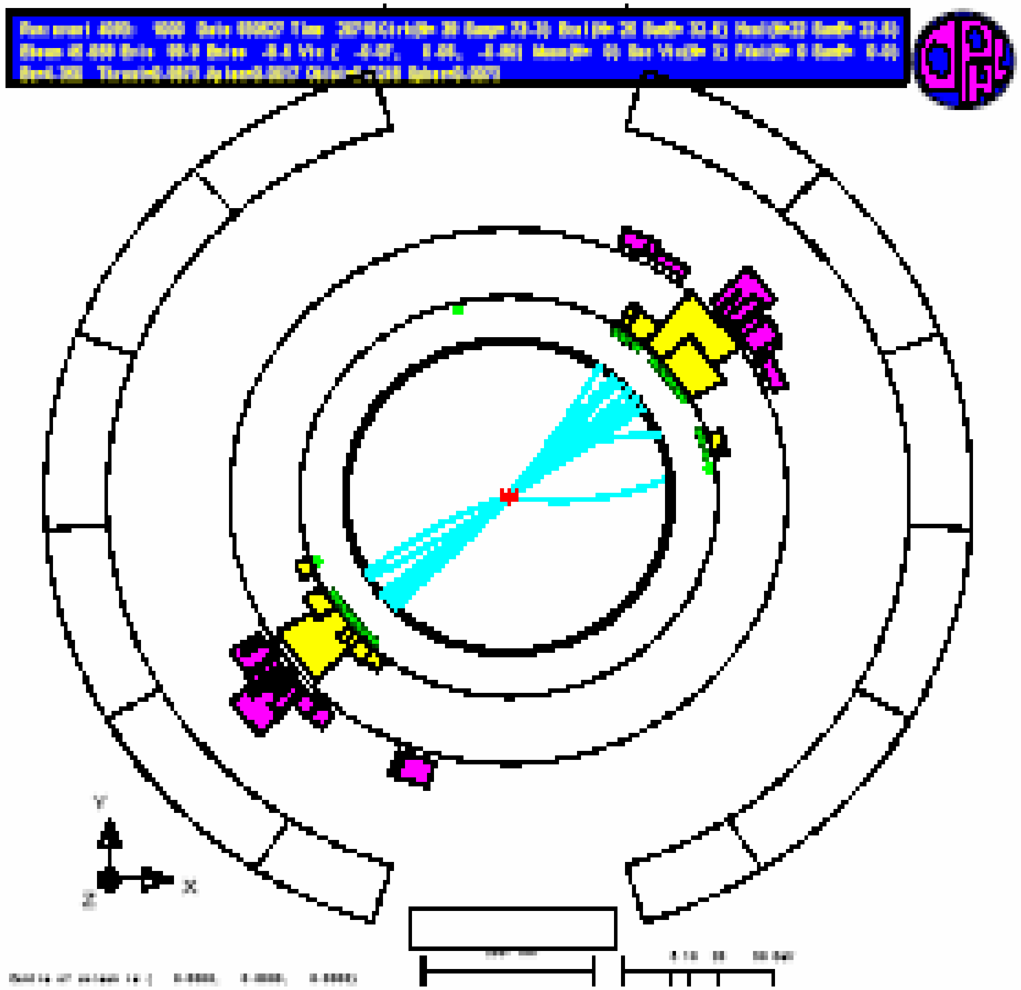
Lowest order ($\mathcal{O}(\alpha^2\alpha_s^0)$):



$e^+e^- \to \text{jet} + \text{jet}$

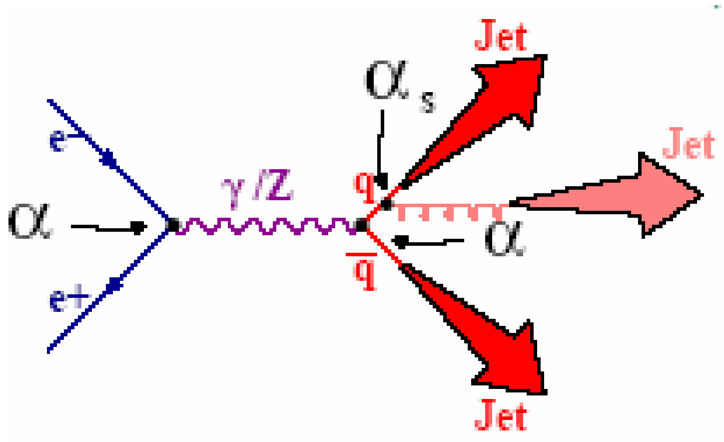
A clean trace of two partons – a pair of quark and antiquark

LEP ($\sqrt{s} = 90 - 205$ GeV)



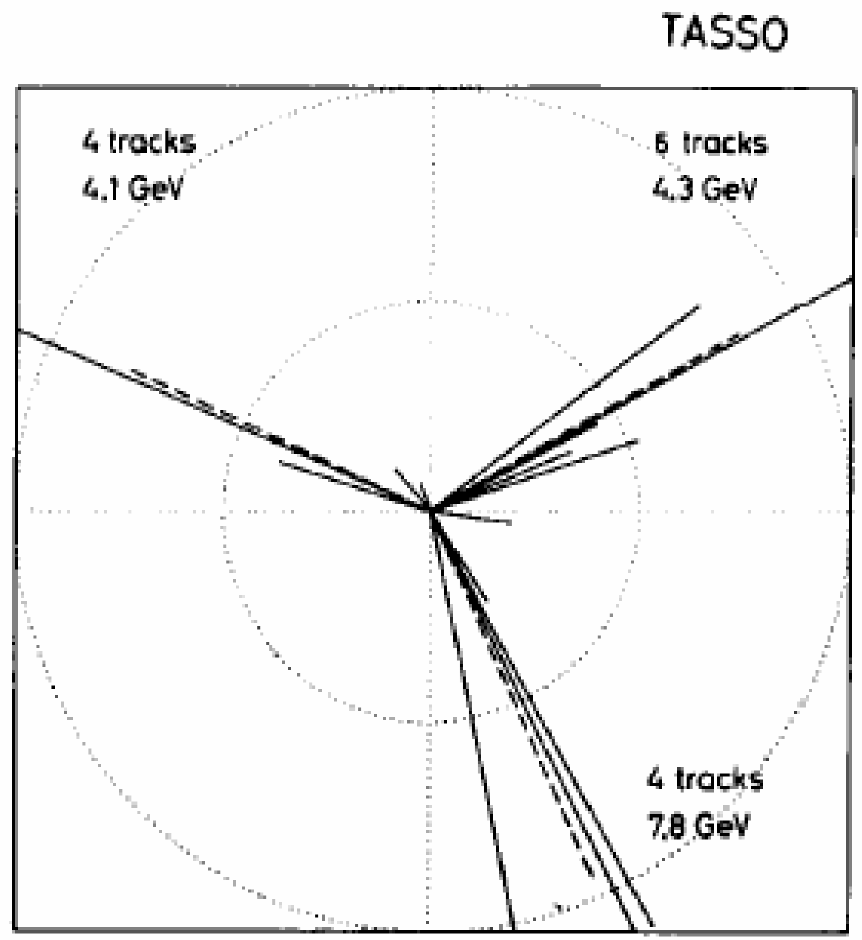
Discovery of a gluon jet

First order in QCD ($\mathcal{O}(\alpha^2\alpha_s^1)$):



PETRA e^+e^- storage ring at DESY:

$E_{c.m.} \gtrsim 15$ GeV



Reputed to be the first three-jet event from TASSO

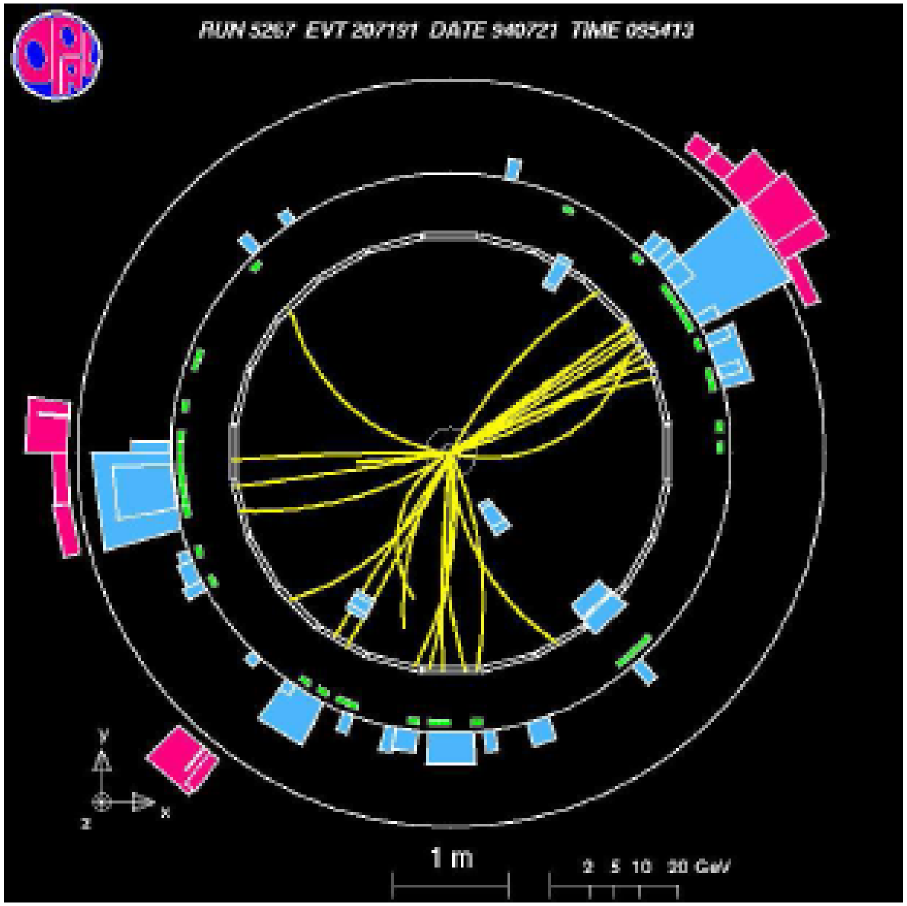
TASSO Collab., Phys. Lett. B86 (1979) 243

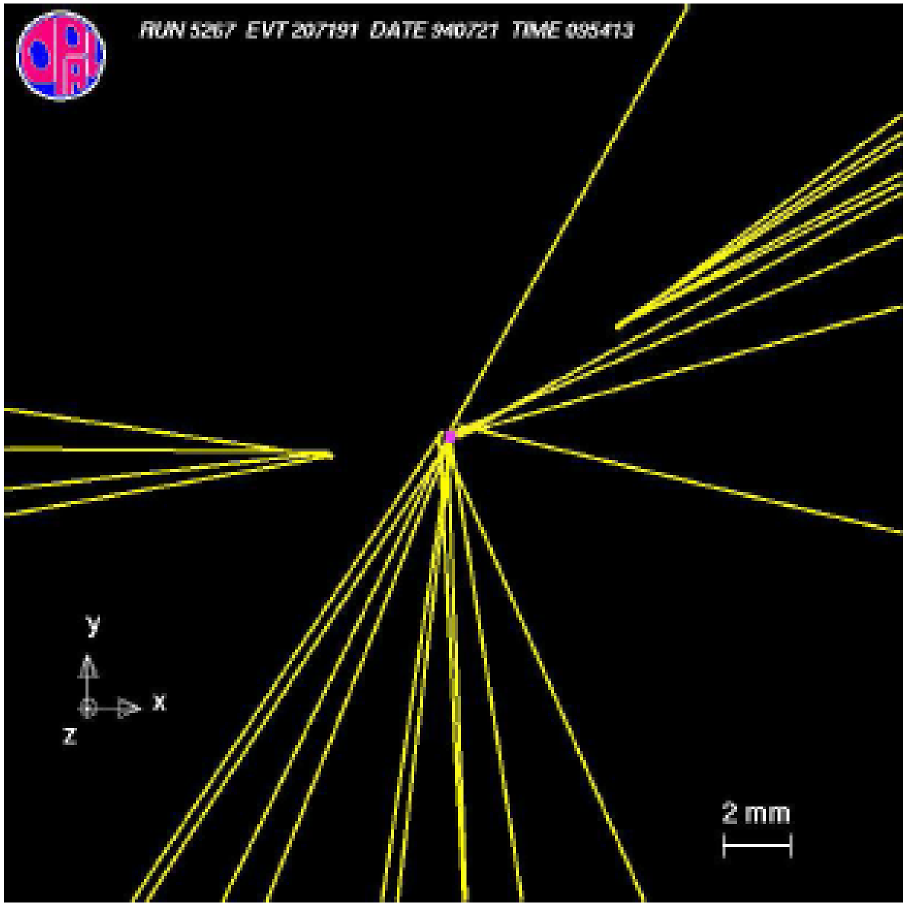
MARK-J Collab., Phys. Rev. Lett. 43 (1979) 830

PLUTO Collab., Phys. Lett. B86 (1979) 418

JADE Collab., Phys. Lett. B91 (1980) 142

Tagged 3-jet event from LEP





Gluon Jet

Basics of jet finding algorithms

- **Recombination jet algorithms:**
  —— **almost universal choice at e+e- colliders**

- Recombination metric: $y_{ij} = \frac{M_{ij}^2}{E_{c.m.}^2}$

  $\longrightarrow$ Combine the particle pair (i, j) with the smallest y_{ij}:

$$(i, j) \to k$$

$\underline{\text{E scheme :}}$ $\quad p_k = p_i + p_j \quad \longrightarrow$ massive jets

$\underline{\text{E}_0 \text{ scheme :}}$ $\quad E_k = E_i + E_j$

$$\vec{p}_k = \frac{\vec{p}_i + \vec{p}_j}{|\vec{p}_i + \vec{p}_j|} E_k \quad \longrightarrow \text{massless jets}$$

- Iterate until all remaining pairs satisfy $y_{ij} > y_{cut}$

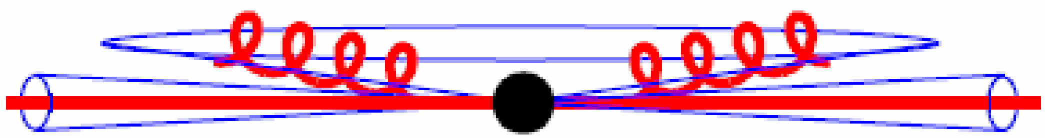
The JADE jet finder

[JADE Collab., Z. Phys. C33 (1986) 23]

⟶ The original recombination jet finder:

- $M_{ij}^2 = 2E_iE_j(1 - \cos\theta_{ij}) \approx$ (invariant mass)^2
- Original version based on the E_0 scheme

Sometimes leads to the formation of "junk jets"



⟶ Two-jet events with ≥ 2 soft, colinear gluons can be classified, unnaturally, as three-jet events

⟶ Prevents re-summation techniques from being applied

The Durham k_T jet finder

[S. Catani et al., Phys. Lett. B269 (1991) 432]

⟶ Introduced to reduce the problem of junk jets

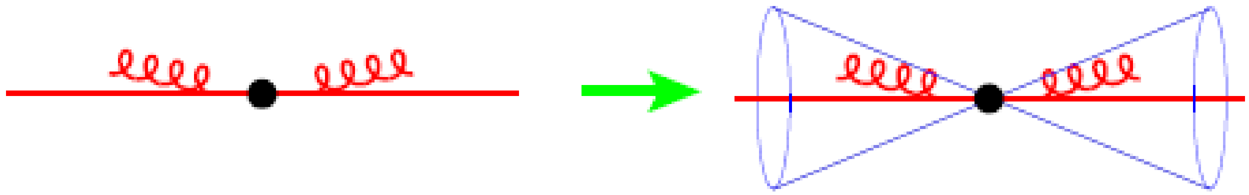
- $M_{ij}^2 = 2\min(E_i^2, E_j^2)(1 - \cos\theta_{ij})$
- E scheme combination of particles: $(i, j) \to k$

⟶ Consider small emission angles θ_{ij}:

$$M_{ij}^2 \approx 2\min(E_i^2, E_j^2)\left[1 - \left(1 - \theta_{ij}^2/2 + \cdots\right)\right]$$
$$\approx \min(E_i^2, E_j^2)\,\theta_{ij}^2 \approx K_\perp^2$$

(min. transverse momenta of one particle w.r.t. the other)

⟶ Soft, colinear radiation is attached to the quark jet(s)

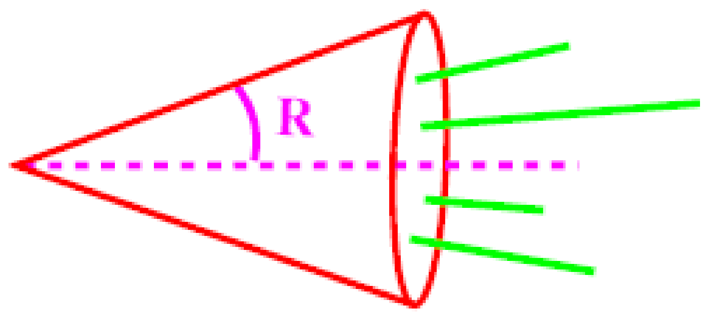


⟶ Permits re-summation

The Cone jet finder

CDF Collab., Phys. Rev. D45, 1448 (1992); OPAL Collab., Z. Phys. C63, 197 (1994)

- Cluster particles within a cone of half angle R into a jet



- Require a minimum visible jet energy: $E_{\text{jet}} \geq \epsilon$

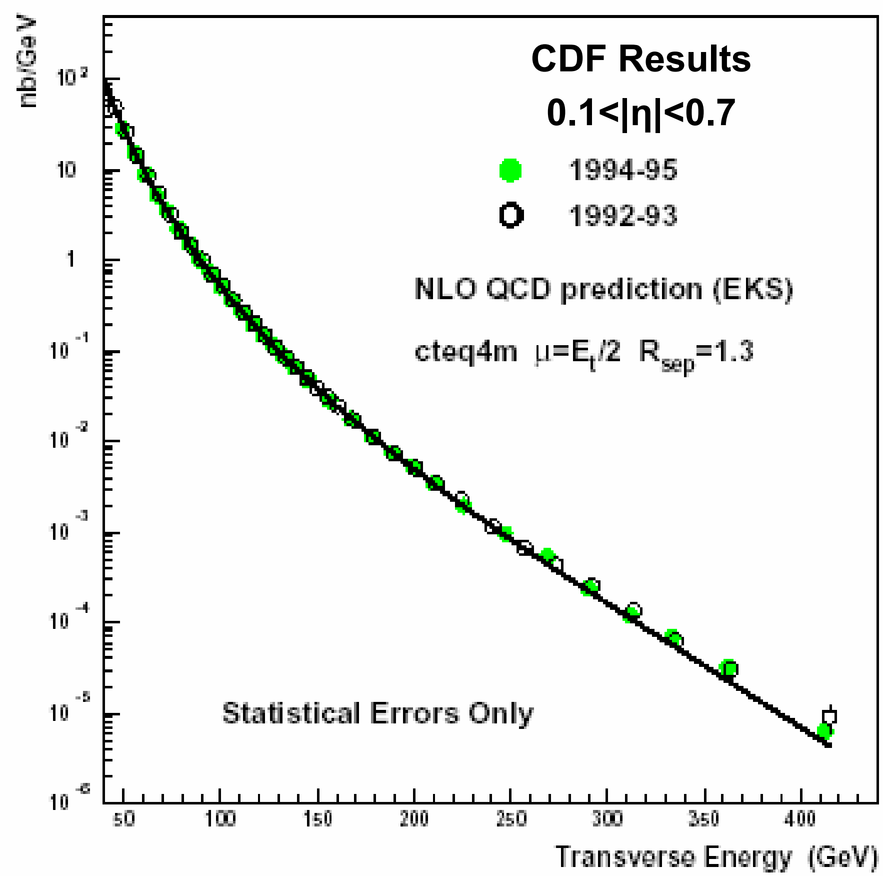
$\longrightarrow$ Two resolution parameters: R and ϵ , as opposed to re-combination algorithms which only have one (y_{cut})

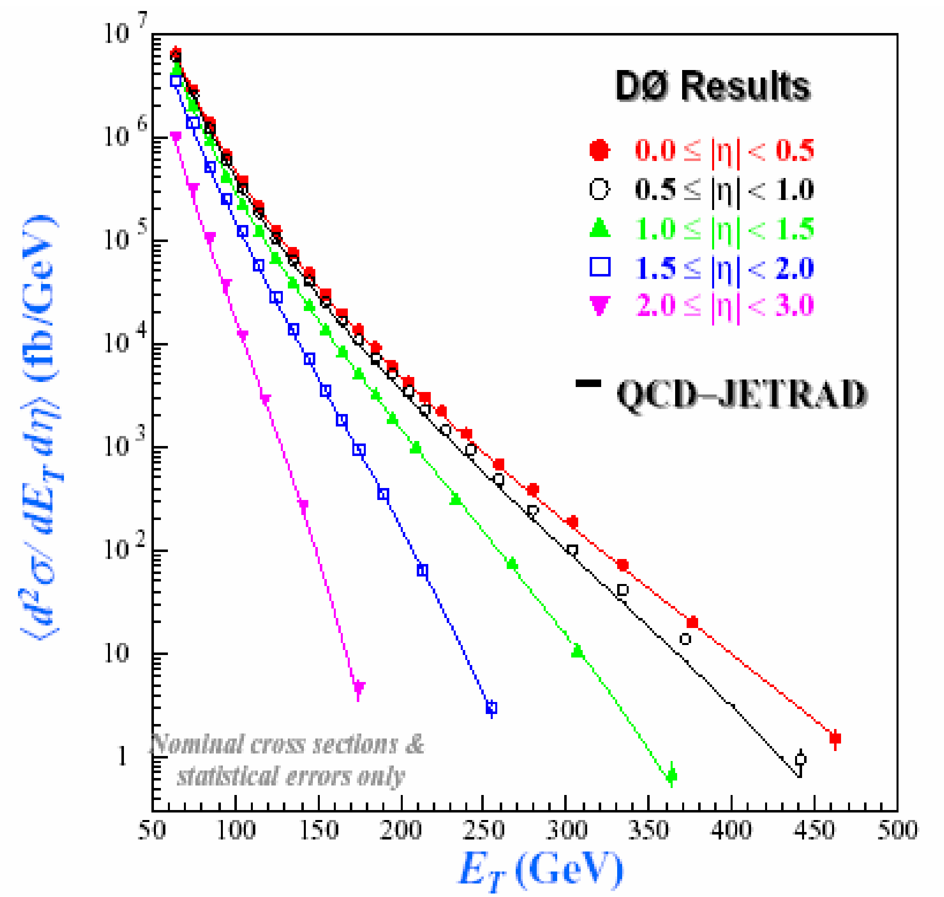
- Eliminate or merge overlapping or redundant jets

$\longrightarrow$ Unlike recombination algorithms, not all particles in an event are necessarily assigned to a jet

Inclusive jet cross section at Tevatron

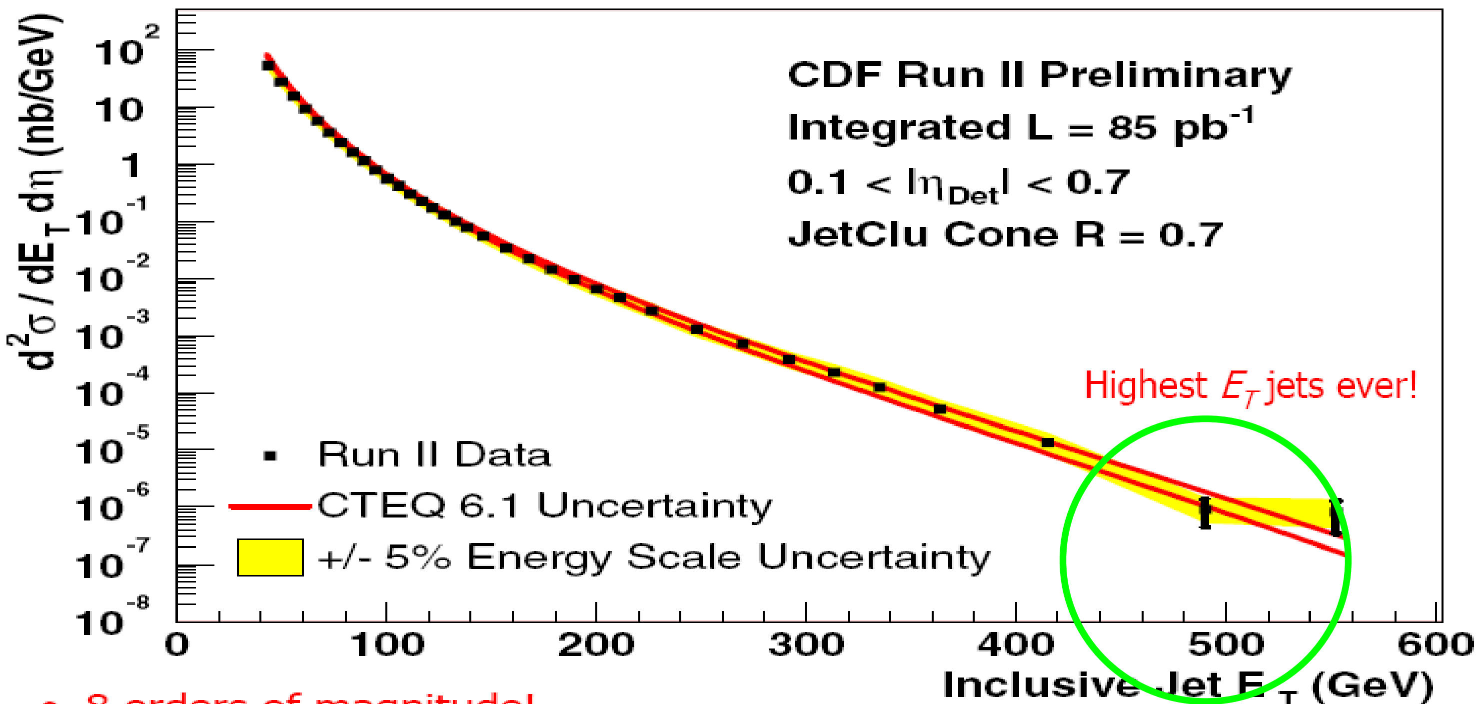
Run – 1b results



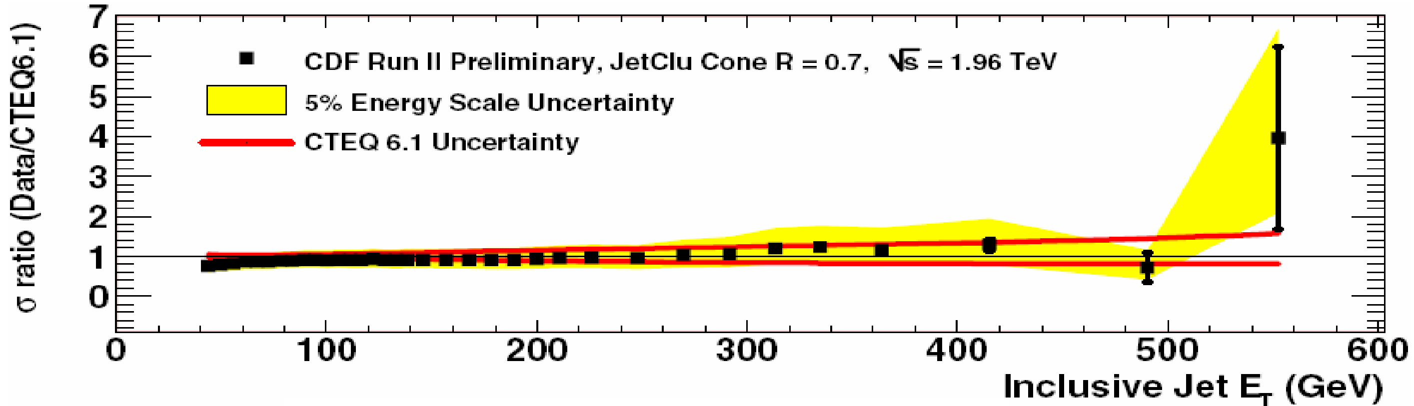


Data and Predictions span 7 orders of magnitude!

Prediction vs CDF Run II data



- 8 orders of magnitude!



Good agreement (within uncertainties)

Infrared safety for jet cross sections

- Jet cross section = inclusive cross section with a phase-space constraint

- For any observable with a phase space constraint, Γ,

$$d\sigma(\Gamma) \equiv \frac{1}{2!}\int d\Omega_2 \frac{d\sigma^{(2)}}{d\Omega_2}\Gamma_2(k_1,k_2) + \frac{1}{3!}\int d\Omega_3 \frac{d\sigma^{(3)}}{d\Omega_3}\Gamma_3(k_1,k_2,k_3) + \ldots + \frac{1}{n!}\int d\Omega_n \frac{d\sigma^{(n)}}{d\Omega_n}\Gamma_n(k_1,k_2,\ldots,k_n) + \ldots$$

Where $\Gamma_n(k_1,k_2,\ldots,k_n)$ are constraint functions and invariant under Interchange of n-particles

- Conditions for IRS of $d\sigma(\Gamma)$:

$$\Gamma_{n+1}\left(k_1,k_2,\ldots,(1-\lambda)k_n^\mu, k_{n+1}^\mu = \lambda k_n^\mu\right) = \Gamma_n\left(k_1,k_2,\ldots,k_n^\mu\right)$$

$$\text{with } 0 \le \lambda \le 1$$

Physical meaning for infrared safety

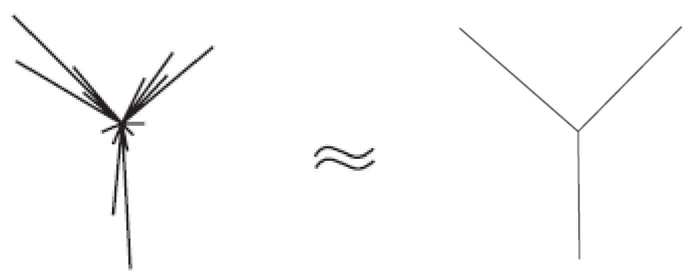
- **Conditions for IRS**

$$\Gamma_{n+1}\left(k_1, k_2, \ldots, (1-\lambda)k_n^\mu, k_{n+1}^\mu = \lambda k_n^\mu\right) = \Gamma_n\left(k_1, k_2, \ldots, k_n^\mu\right)$$

$$\text{with } \ 0 \le \lambda \le 1$$

- **Physical meaning:**

Measurement cannot distinguish a state
with an additional zero/collinar momentum parton
from a state without the parton

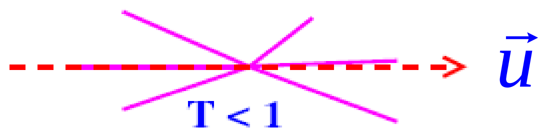


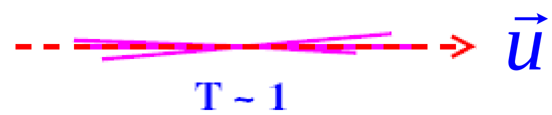
Special case: $\Gamma_n\left(k_1, k_2, \ldots, k_n\right) = 1 \ \text{for all } n \quad \Rightarrow \quad \sigma^{(\text{tot})}$

Thrust distribution

- **Thrust axis:** $\vec{u}$

$$T_n\left(p_1^\mu, p_2^\mu, \ldots, p_n^\mu\right) = \max_{\vec{u}} \left(\frac{\sum_{i=1}^{n} \vec{p}_i \cdot \vec{u}}{\sum_{i=1}^{n} \left|\vec{p}_i\right|} \right)$$





- **Phase space constraint:**

$$\Gamma_n\left(p_1^\mu, p_2^\mu, \ldots, p_n^\mu\right) = \delta\left(T - T_n\left(p_1^\mu, p_2^\mu, \ldots, p_n^\mu\right)\right)$$

- Contribution from p=0 particles drops out the sum
- Replace two collinear particles by one particle does not change the thrust

$$|(1-\lambda)\,\vec{p}_n \cdot \vec{u}| + |\lambda\,\vec{p}_n \cdot \vec{u}| = |\vec{p}_n \cdot \vec{u}|$$

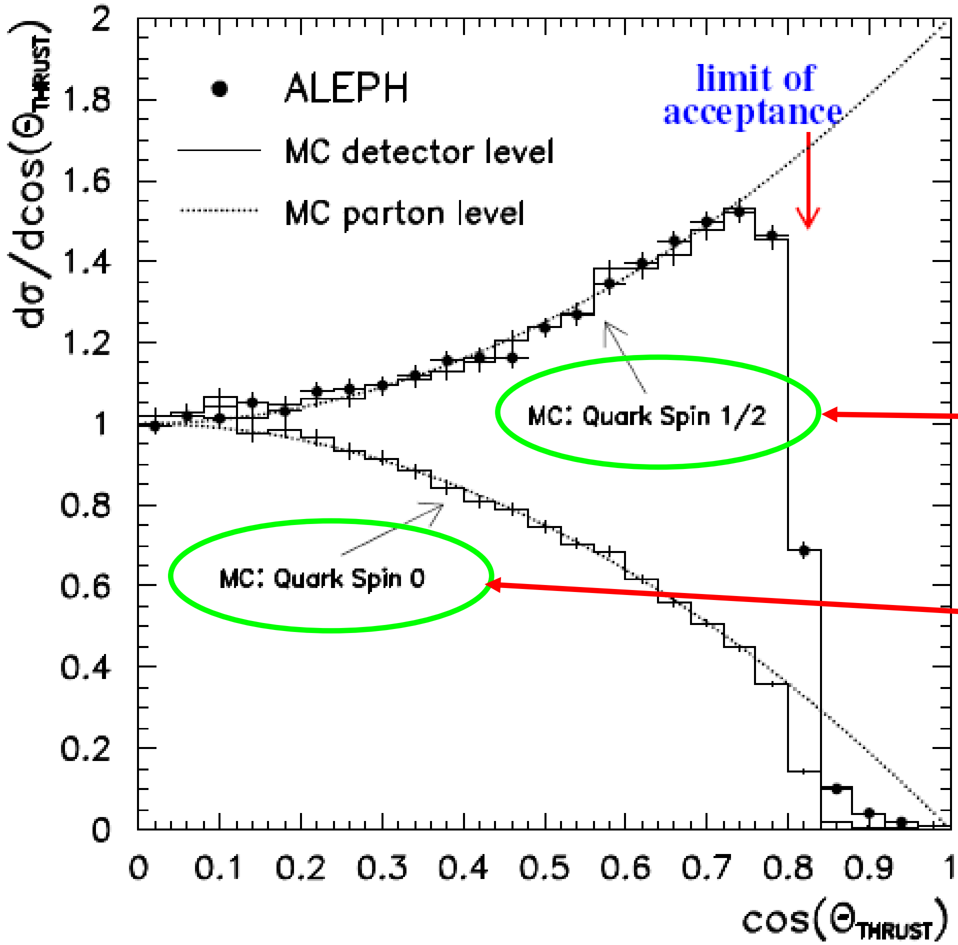
and

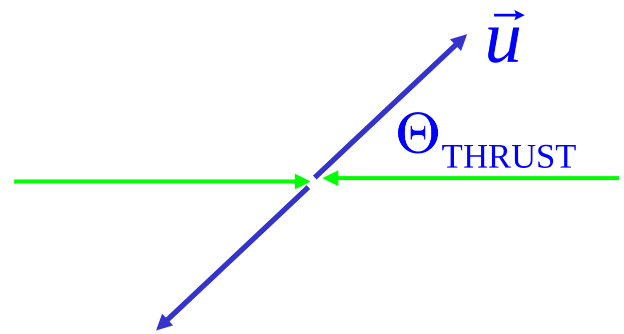
$$|(1-\lambda)\,\vec{p}_n| + |\lambda\,\vec{p}_n| = |\vec{p}_n|$$

Another test of quark spin

- **Angle between the thrust axis and the beam axis:**

[ALEPH Collab., Phys. Rep. 294 (1998) 1]





At LO:

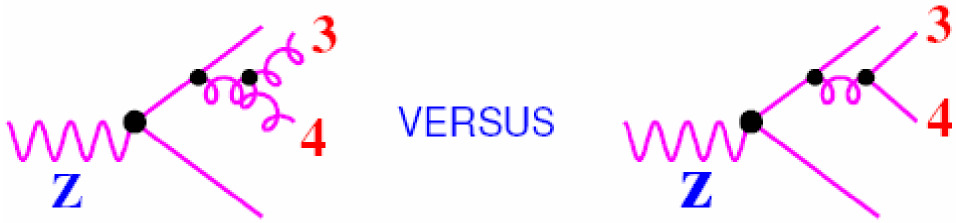
$1+\cos^2\Theta_{THRUST}$

$1-\cos^2\Theta_{THRUST}$

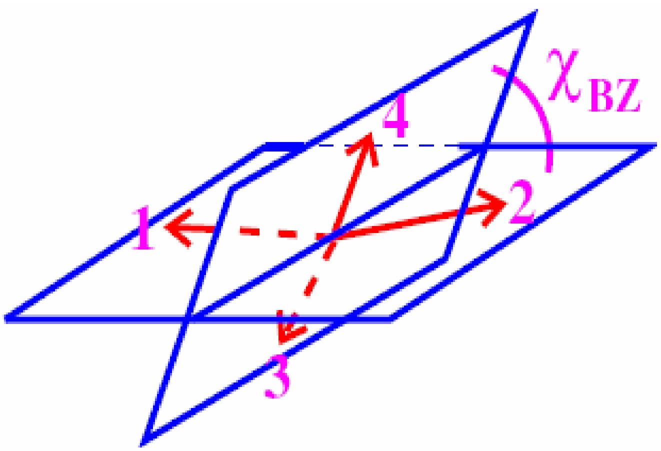
Another test of $SU(3)_{color}$

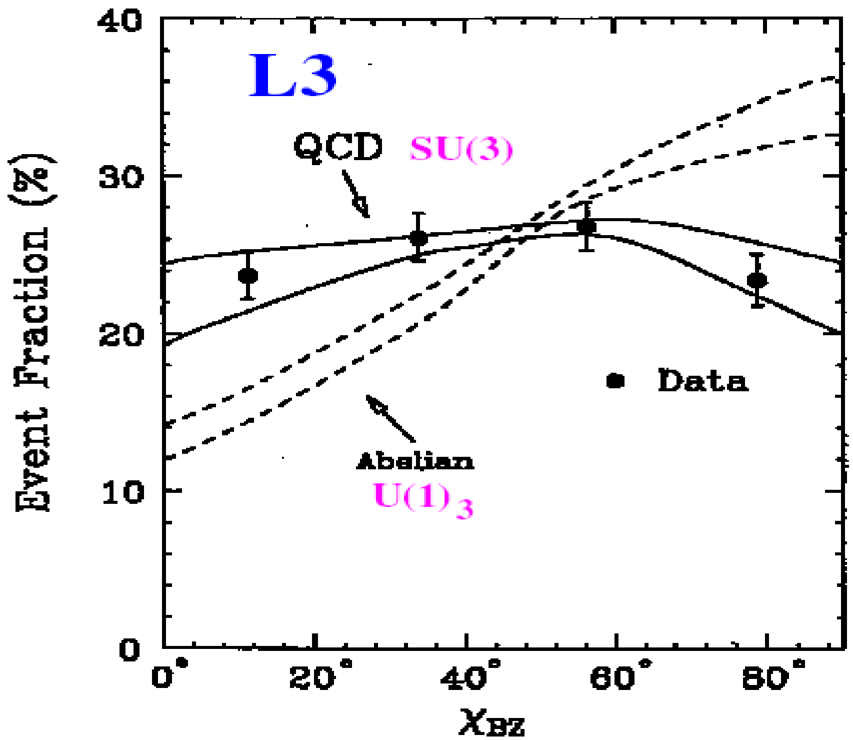
- **Select 4-jet events:** $E_1 > E_2 > E_3 > E_4$

jet-3 and jet-4 are more likely from radiation



- **Bengtsson-Zerwas angle:**





Question

Does perturbative QCD work for cross sections with identified hadrons?

Facts:

Typical hadronic scale: $1/R \sim 1\ \text{fm}^{-1} \sim \Lambda_{QCD}$

Energy exchange in hard collisions: $Q >> \Lambda_{QCD}$

pQCD works at $\alpha_s(Q)$, but not at $\alpha_s(1/R)$

Answer:

Perturbative QCD factorization

Summary

- QCD is a SU(3) color non-Abelian gauge theory of quark and gluon fields
- QCD perturbation theory works at high energy because of the asymptotic freedom
- Perturbative QCD calculations make sense only for infrared safe (IRS) quantities
- Jets in high energy collisions provide us the “trace” of energetic quarks and gluons
- can we actually “see” and “count” the quarks and gluons?