Introduction to Perturbative QCD

Lecture 1

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❑ The Goal:

To understand hadron structure, and strong interaction dynamics in terms of Quantum Chromodynamics (QCD)

❑ The Plan:

From the Parton Model to QCD and pQCD
One lecture

Purely infrared safe observables in pQCD
One lecture

Cross sections with identified hadrons in pQCD
Two lectures

Outline for Lecture 1

- ❑ Nucleons to Quarks
- ❑ Deep Inelastic Scattering (DIS)
- ❑ The Parton Model
- ❑ Extensions of Parton Model beyond DIS
- ❑ Quantum Chromodynamics (QCD)
- ❑ Asymptotic freedom and perturbative QCD

Excellent resource – CTEQ summer school website

http://www.phys.psu.edu/~cteq

Nucleons to Quarks

❑ Protons, Neutrons, and Pions

$$p\begin{cases} m = 938.3\ \text{MeV} \\ S = 1/2 \\ I_3 = +1/2 \end{cases} \qquad n\begin{cases} m = 939.6\ \text{MeV} \\ S = 1/2 \\ I_3 = -1/2 \end{cases}$$

Isospin doublet $\quad N = \begin{pmatrix} p \\ n \end{pmatrix}$

$$\pi^{\pm}\begin{cases} m = 139.6\ \text{MeV} \\ S = 0 \\ I_3 = \pm 1 \end{cases} \qquad \pi^{0}\begin{cases} m = 135.0\ \text{MeV} \\ S = 0 \\ I_3 = 0 \end{cases}$$

Isospin triplet $\quad \pi = \begin{pmatrix} \pi^+ \\ \pi^0 \\ \pi^- \end{pmatrix}$

❑ "Historic" – π as $N\bar{N}$ bound states

$$\pi^+ = \left(p\bar{n}\right),\quad \pi^- = \left(n\bar{p}\right),\quad \pi^0 = \frac{1}{\sqrt{2}}\left(p\bar{p} + n\bar{n}\right)$$

Fermi and Yang, 1952; Nambu and Jona-Lasinio, 1960 (dynamics)

Nucleons not point-like spin ½ Dirac particles

Proton magnetic moment: $g_p \neq 2$

Neutron magnetic moment: $g_n \neq 0$

"Modern" – π, N common substructure: quarks

→ **Quark Model** – Gell Mann, Zweig, 1964

Quarks:

$$u \begin{cases} Q = 2/3e \\ S = 1/2 \\ I_3 = +1/2 \end{cases} \quad d \begin{cases} Q = -1/3e \\ S = 1/2 \\ I_3 = -1/2 \end{cases} \quad s \begin{cases} Q = -1/3e \\ S = 1/2 \\ I_3 = 0 \end{cases}$$

$$\pi^+ = (u\bar{d}), \quad \pi^- = (d\bar{u}), \quad \pi^0 = \frac{1}{\sqrt{2}}(u\bar{u} + d\bar{d})$$

$$p = (uud), \quad n = (udd), \quad K^+ = (u\bar{s}), \ldots, \Delta^{++} = (uuu), \ldots$$

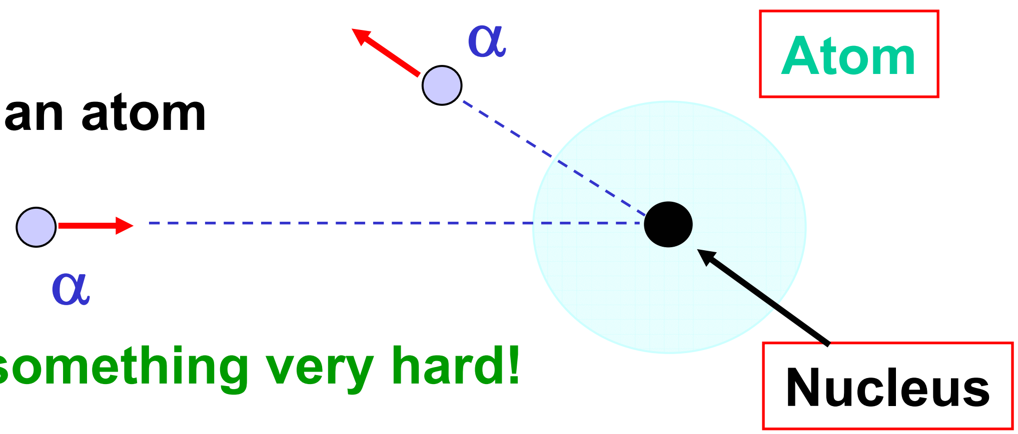
Magnetic moment: $\mu_p/\mu_n = g_p/g_n = -3/2$ (good to %)

But, need a new quantum number – color and the dynamics! Han, Nambu, 1965

How to "see" substructure of a nucleon?

- ❑ **Rutherford experiment:**

  – to see the substructure of an atom



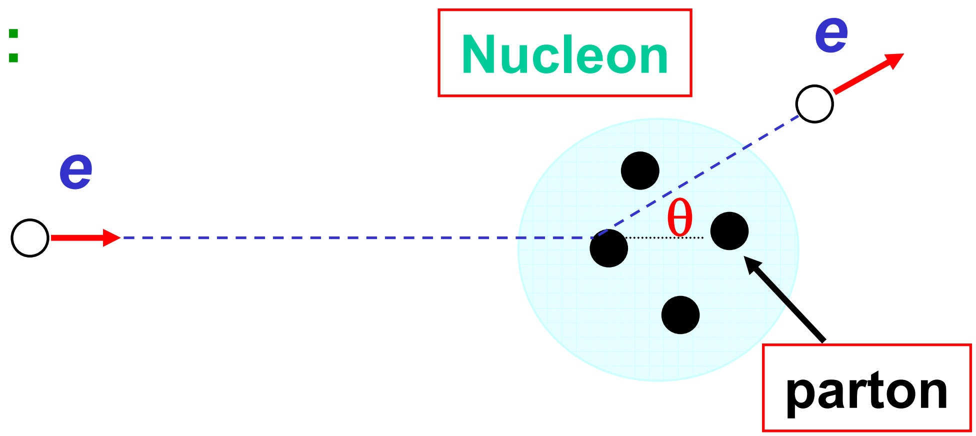
High energy α bounce off something very hard!

➡ **Discovery of nucleus inside an atom**

- ❑ **SLAC experiment (1969):**

  Lepton-nucleon deeply inelastic scattering (DIS)

Scattering information on the θ-distribution



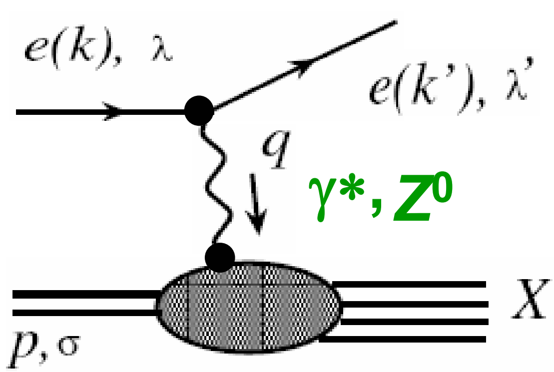
➡ **Discovery of the point-like spin-1/2 "partons"**

Callan-Gross relation

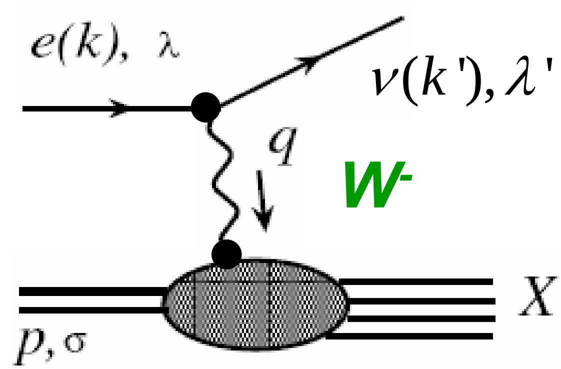
Lepton-hadron DIS

❑ Process: $e(k,\lambda)+P(p,\sigma)\rightarrow e(k',\lambda')+X$

Neutral current (NC)



Charged current (CC)



❑ Kinematics:

- ❖ **4-momentum transfer:** $Q^2=-q^2$
- ❖ **Bjorken variable:** $x_B=\frac{Q^2}{2p\cdot q}$
- ❖ **Squared CMS energy:** $s=(p+k)^2=\frac{Q^2}{x_B y}$
- ❖ **Inelasticity:** $y=\frac{p\cdot q}{p\cdot k}$
- ❖ **Final-state hadronic mass:** $W^2=(p+q)^2\approx\frac{Q^2}{x_B}(1-x_B)$

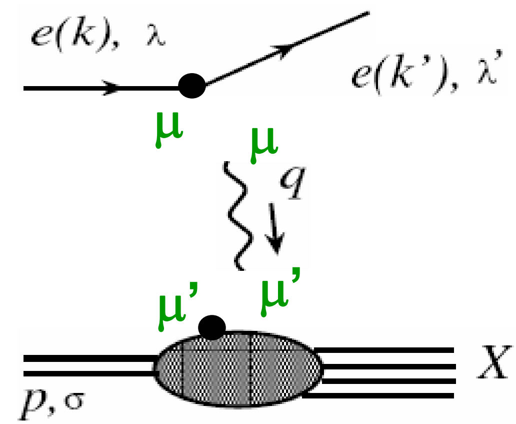
Lepton-hadron DIS – general analysis

❑ Scattering amplitude:

$$M(\lambda,\lambda';\sigma,q) = \bar{u}_{\lambda'}(k')\left[-ie\gamma_{\mu}\right]u_{\lambda}(k)$$

$$* \left(\frac{i}{q^2}\right)\left(-g^{\mu\mu'}\right)$$

$$* \left\langle X\left|eJ^{em}_{\mu'}(0)\right|p,\sigma\right\rangle$$



❑ Cross section:

$$d\sigma^{\rm DIS} = \frac{1}{2s}\left(\frac{1}{2}\right)^2 \sum_X \sum_{\lambda,\lambda',\sigma} \left|M(\lambda,\lambda';\sigma,q)\right|^2 \left[\prod_{i=1}^{X} \frac{d^3 l_i}{(2\pi)^3 2E_i}\right] \frac{d^3 k'}{(2\pi)^3 2E'} (2\pi)^4 \delta^4\left(\sum_{i=1}^{X} l_i + k' - p - k\right)$$

$$\Longrightarrow \quad E'\frac{d\sigma^{\rm DIS}}{d^3k'} = \frac{1}{2s}\left(\frac{1}{Q^2}\right)^2 L^{\mu\nu}(k,k')W_{\mu\nu}(q,p)$$

❑ Leptonic tensor:

– known from QED

$$L^{\mu\nu}(k,k') = \frac{e^2}{2\pi^2}\left(k^{\mu}k'^{\nu} + k^{\nu}k'^{\mu} - k\cdot k' g^{\mu\nu}\right)$$

❑ Hadronic tensor (No QCD has been used):

$$W_{\mu\nu}(q,p) = \frac{1}{4\pi}\left\{\frac{1}{2}\sum_{\sigma}\int d^4z\ \mathrm{e}^{iq\cdot z}\ \left\langle p,\sigma\left| J_\mu^\dagger(z) J_\nu(0) \right| p,\sigma\right\rangle\right\}$$

❑ Structure functions:

- ❖ **Parity invariance (EM current)** ⟶ $W_{\mu\nu} = W_{\nu\mu}$ sysmetric for spin avg.
- ❖ **Time-reversal invariance** ⟶ $W_{\mu\nu} = W^*_{\mu\nu}$ real
- ❖ **Current conservation** ⟶ $q^\mu W_{\mu\nu} = q^\nu W_{\mu\nu} = 0$

$$W_{\mu\nu} = -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right) F_1\left(x_B, Q^2\right) + \frac{1}{p\cdot q}\left(p_\mu - q_\mu \frac{p\cdot q}{q^2}\right)\left(p_\nu - q_\nu \frac{p\cdot q}{q^2}\right) F_2\left(x_B, Q^2\right)$$

Reduced to two dimensionless scalar structure functions for spin-avgeraged DIS

Two more structure functions for spin-dependent DIS

Measure cross sections ⇔ extraction of structure functions

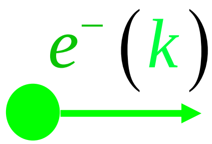
Note: No explicit QCD was used in above derivation!

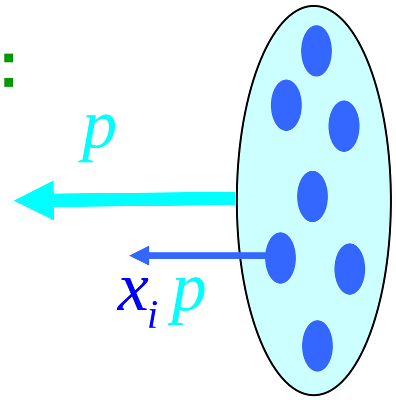
The Parton Model

Feynman, 1969, 1972

- **Before the collision:**

in e⁻–parton cm frame:





$$t_{collision} \sim \frac{1}{Q} \ll \text{fm} \sim t_{hadron}$$

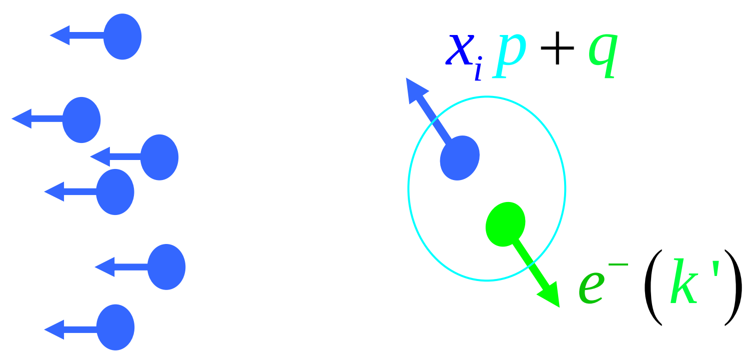
$$0 \le x_i \le 1$$

$$\sum_i x_i = 1$$

Lorentz contracted
Time dilated
Effectively frozen

- **After the collision:**

fragments



$$\left(x_i\,p + q\right)^2 \approx 0$$

$$\left[\,i.e., \ll \left|q^2\right|\,\right]$$

elastic collision

"Deeply inelastic scattering"

Basic Parton Model Relation

$$\sigma_{eh}^{\mathrm{DIS}}(p,q) = \sum_{partons-f} \int_0^1 dx\, \hat{\sigma}_{ef}^{\mathrm{el}}(xp,q)\ \phi_f(x)$$

where

$\sigma_{eh}^{\mathrm{DIS}}(p,q)$ **DIS cross section for hadron:** $h(p)$

$\hat{\sigma}_{ef}^{\mathrm{el}}(xp,q)$ **Elastic cross section for parton:** $f(xp)$

$\phi_f(x)$ **Probability for** f **to have** xp **- PDF**

Inelastic hadronic cross section	$=$	Partonic elastic cross section	$\otimes$	Probability for $p_f = xp$

Nontrivial assumption:

Quantum mechanical incoherence between the large *q* scattering and the partonic distribution

Structure Functions in Parton Model

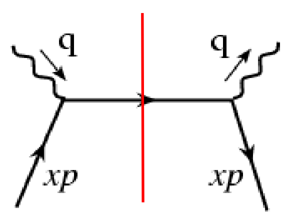
Recall:

$$E'\frac{d\sigma_{eh}^{\text{DIS}}}{d^3k'} = \frac{1}{2s}\left(\frac{1}{Q^2}\right)^2 L^{\mu\nu}(k,k')W_{\mu\nu}(q,p)$$

PM formula:

$$W_{\mu\nu}(q,p) = \sum_{partons-f}\int_0^1 dx\left[\frac{1}{x}\hat{W}_{\mu\nu}^{\text{el}}(q,xp)\right]\phi_f(x)$$

$$\hat{W}_{\mu\nu}^{\text{el}}(q,xp) = \sum_f e_f^2 \frac{1}{4\pi}\frac{1}{2}\text{Tr}\left[\gamma_\mu\gamma\cdot(xp+q)\gamma_\nu\gamma\cdot(xp)\right](2\pi)\delta\left((xp+q)^2\right)$$



$$= -\left(g_{\mu\nu} - \frac{q_\mu q_\nu}{q^2}\right)\left[e_f^2\,\frac{1}{2}\delta\left(1-\left(\frac{x_B}{x}\right)\right)\right]$$

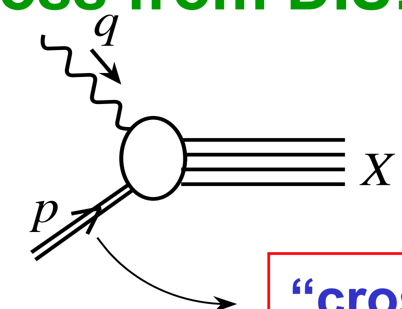
$$+\frac{1}{p\cdot q}\left(p_\mu - q_\mu\frac{p\cdot q}{q^2}\right)\left(p_\nu - q_\nu\frac{p\cdot q}{q^2}\right)\left[e_f^2\; x\;\delta\left(1-\left(\frac{x_B}{x}\right)\right)\right]$$

$$\Longrightarrow F_2(x_B,Q^2) = \sum_f e_f^2\; x_B\;\phi_f(x_B) = 2x_B F_1(x_B,Q^2)$$

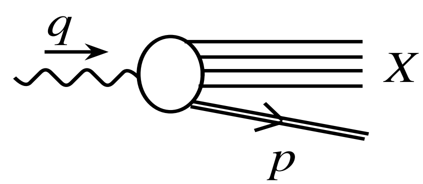
- ❑ **Callan-Gross Relation → spin ½ parton**
- ❑ **Bjorken scaling → Q^2- independent universal PDFs**

Fragmentation Functions in PM

- **Cross from DIS:**

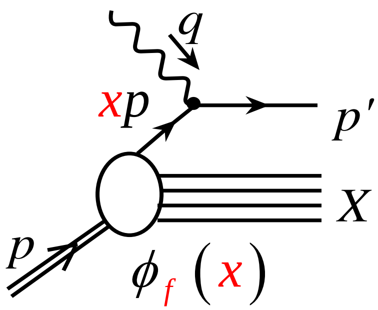


“crossing”

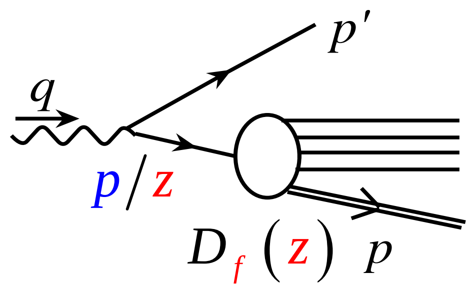


Single particle inclusive (1PI)

- **Cross from Parton Model:**



Parton distribution



Fragmentation function

- **PM formula for 1PI:**

$$\sigma_h^{1PI}(p,q) = \sum_{partons-f} \int_0^1 dz\, \hat{\sigma}_f^{1PI}(p/z, q)\, D_f(z)$$

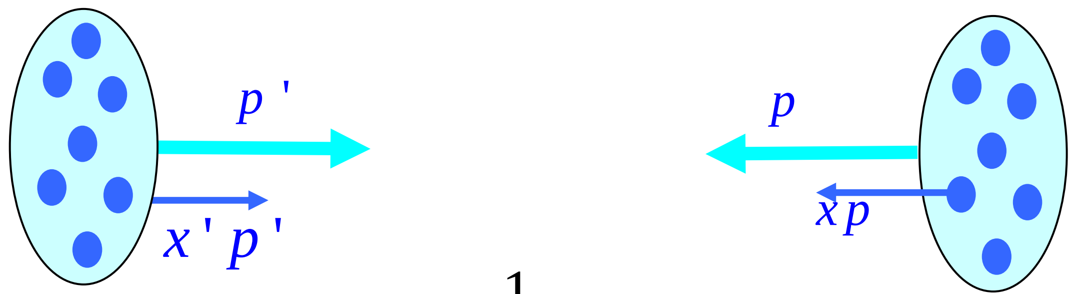
Drell-Yan Dilepton Production in PM

S.D. Drell and T.-M.Yan, PRL 25, 316 (1970)

- **Drell-Yan Process:**

$$h(p)+h'(p')\rightarrow \ell^+\ell^-(q)+X \qquad \text{with } q^2=Q^2$$

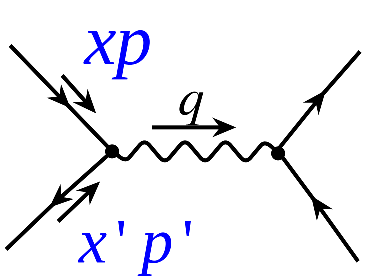
- **PM picture:**



$$t_{collision}\sim\frac{1}{Q}\ll \text{fm}\sim t_{hadron}$$

- **PM formula:**

$$\frac{d\sigma^{DY}_{hh'}(p,p',q)}{dQ^2}=\sum_{f,f'}\int_0^1 dx\int_0^1 dx'\,\phi_f(x)\,\frac{d\hat{\sigma}^{el}_{ff'}(xp,x'p',q)}{dQ^2}\,\phi_{f'}(x')$$



$$\frac{d\hat{\sigma}^{el}_{ff'}(xp,x'p',q)}{dQ^2}=e_f^2\delta_{ff'}\,\frac{1}{3}\,\frac{4\pi\alpha^2_{em}}{3Q^2}\,\frac{1}{xx's}\,\delta\left(1-\left(\frac{Q^2}{xx's}\right)\right)$$

$$\text{ColorFactor}=\left(\frac{1}{3}\right)^2\sum_{i,j=1}^{3}\delta_{ij}\delta_{ji}=\frac{1}{3}$$

Need to Improve the PM

- **Total momentum carried by the partons:**

$$F_q \equiv \sum_f \int_0^1 dx\, x\, \phi_f(x) \ \sim\ 0.5$$

missing momentum

→ **particles not directly interact with photon (or EM charge)** → **the gluon**

- **Scaling violation**

→ **Q-dependence of structure functions?**

- **Drell-Yan cross section:**

$$K\Big|_{\mathrm{Exp/Thy}} = \sigma_{hh'}^{\mathrm{DY}}\Big|_{\mathrm{Exp}} \Big/ \sigma_{hh'}^{\mathrm{DY}}\Big|_{\mathrm{Thy}} \geq 2$$

- **...**

Need a better dynamical theory!

Quantum Chromodynamics (QCD)

❑ **Known Fundamental Interactions:**

AdS/CFT correspondence

- "Strong" – QCD
- Weak
- Electromagnetic - QED
- …
- Gravity

Weak + Electromagnetic → **Electro – Weak**

Strong + Electro – Weak → **Standard Model**

❑ **QCD – stands as a very solid building block of the SM:**

Unbroken SU(3) color gauge symmetry

Asymptotic freedom at high energy

Success of QCD perturbation theory

Nonperturbative results from Lattice calculations

… Not many surprises so far

QCD as a field theory

- **Fields:**

$\psi_i^f(x)$ Quark fields, Dirac fermions (like *electron*)
Color triplet: $i = 1,2,3 = N_C$
Flavor: $f = u,d,s,c,b,t$

$A_{\mu,a}(x)$ Gluon fields, spin-1 vector field (like *photon*)
Color octet: $a = 1,2,\ldots,8 = N_C^2 - 1$

- **Lagrangian density:**

$$L_{QCD}(\psi, A) = \sum_f \overline{\psi}_i^f \left[\left(i\partial_\mu - gA_{\mu,a}(t_a)_{ij} \right)\gamma^\mu - m_f \right]\psi_i^f$$
$$-\frac{1}{4}\left[\partial_\mu A_{\nu,a} - \partial_\nu A_{\mu,a} - gC_{abc}A_{\mu,b}A_{\nu,c}\right]^2$$

+ *gauge fixing* + *ghost terms*

Color matrix: $[t_a, t_b] = iC_{abc}t_c$

❑ Gauge invariance:

$$\psi_i \to \psi'_j = U_{ji}(x)\psi_i$$

$$A_\mu \to A_\mu' = U(x)A_\mu U^{-1}(x) + \frac{i}{g}\left[\partial_\mu U(x)\right]U^{-1}(x)$$

where $A_\mu = A_{\mu,a}t_a$, $U_{ij}(x)$ unitary [det = 1, SU(3)]

❑ Gauge fixing:

$$\mathcal{L}_{gauge} = -\frac{\lambda}{2}\left(\partial_\mu A_a^\mu\right)\left(\partial_\nu A_a^\nu\right)$$

Allow us to define a propagator:

$$G_{\mu\nu}(k)_{ab} = \frac{\delta_{ab}}{k^2}\left[-g_{\mu\nu} + \frac{k_\mu k_\nu}{k^2}\left(1 - \frac{1}{\lambda}\right)\right]$$

with $\lambda = 1$ **Feynman gauge**

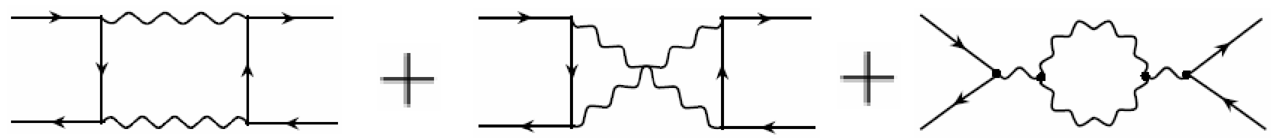
❑ Ghost:

ghost fields

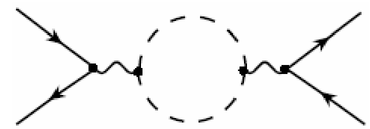
$$\mathcal{L}_{ghost} = (\partial_\mu \bar{\eta}_a)\,(\partial^\mu \eta_a - g C_{abc} A_b^\mu \eta_c)$$

so that optical theorem (and hence unitarity) may be respected:

2 Im [



+ ... +



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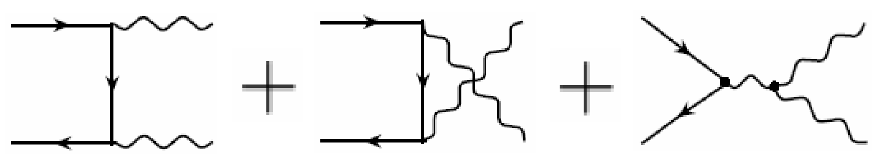
$= \sum$ |



$|^2$

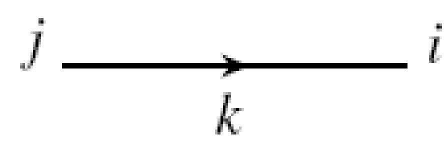
Sum over all physical polarizations

Fail without the ghost loop

Feynman rules

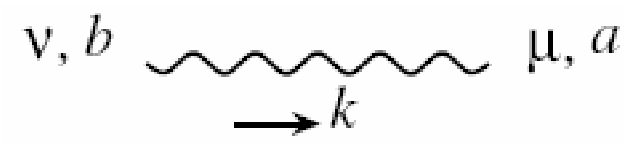
❑ Propagators:

Quark:



$$\frac{i}{\gamma \cdot k - m}\delta_{ij}$$

Gluon:



$$\frac{i\delta_{ab}}{k^2}\left[-g_{\mu\nu} + \frac{k_\mu \nu}{k^2}\left(1 - \frac{1}{\lambda}\right)\right]$$

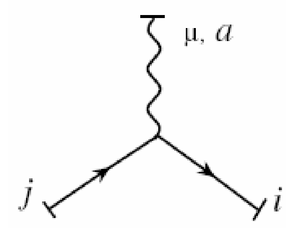
for covariant gauge

Ghost:

$$\frac{i\delta_{ab}}{k^2}$$

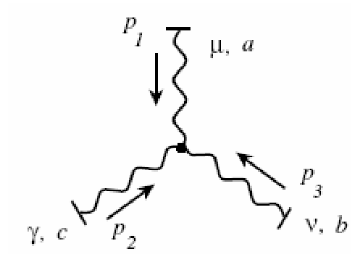
❑ Interactions:

$$-g\bar{\psi}\gamma^{\mu}A_{\mu,a}t_a\psi$$



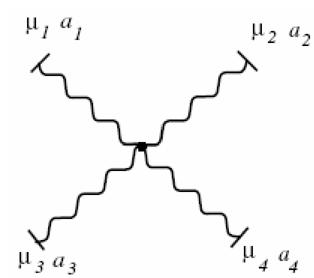
$$-ig(t_a)_{ij}\gamma_{\mu}$$

$$\frac{1}{2}gC_{abc}(\partial_{\mu}A_{\nu,a} - \partial_{\nu}A_{\mu,a})A_b^{\mu}A_c^{\nu}$$



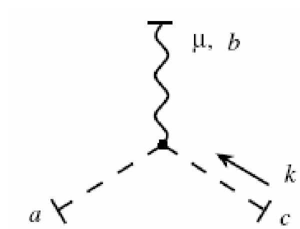
$$-gC_{abc}\left[g_{\mu\nu}(p_1 - p_2)_{\gamma} + g_{\nu\gamma}(p_2 - p_3)_{\mu} + g_{\gamma\mu}(p_3 - p_1)_{\nu}\right]$$

$$-\frac{g^2}{4}C_{abc}C_{ab'c'} * A_b^{\mu}A_c^{\nu}A_{\mu,b'}A_{\nu,c'}$$



$$-ig^2\left[C_{ea_1a_2}C_{ea_3a_4} * (g_{\mu_1\mu_3}g_{\mu_2\mu_4} - g_{\mu_1\mu_4}g_{\mu_2\mu_3}) + ...\right]$$

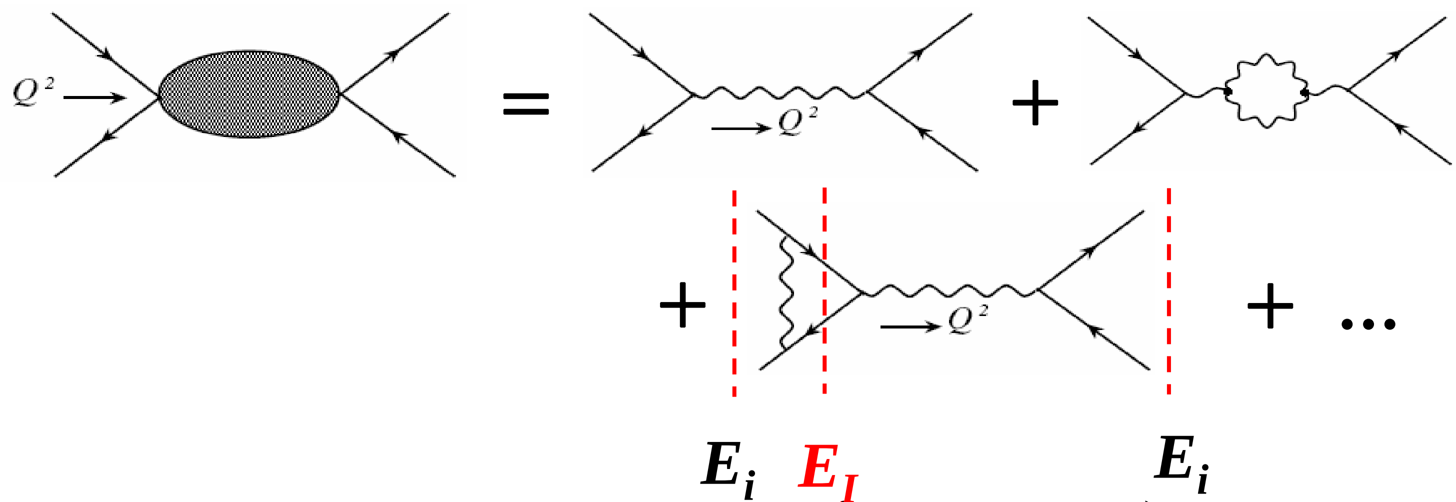
$$\partial_{\mu}\bar{\eta}_a\,(gC_{abc}A_b^{\mu})\,\eta_c$$



$$gC_{abc}k_{\mu}$$

Renormalization in QCD

- ❑ **Scattering amplitude:**



$$= \int \langle PS \rangle_I \left(\frac{1}{E_i - E_I} + \ldots \right) + \ldots \Rightarrow \infty$$

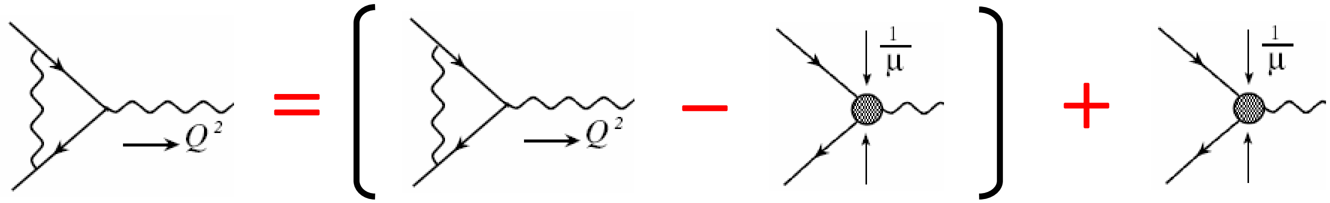
UV divergence ➡ **"Sum" over states of "high mass"**

Uncertainty principle: high mass states = "Local" interaction

No experiment has an infinite resolution!

❑ Renormalization:

- ❖ UV divergence due to “high mass” states
- ❖ Experiments cannot resolve the details of these states

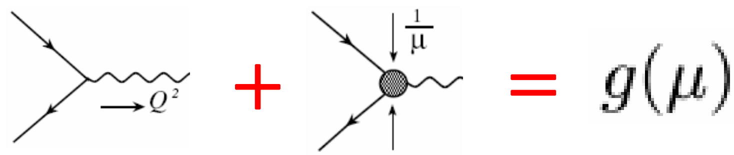


“Low mass” state

“High mass” states

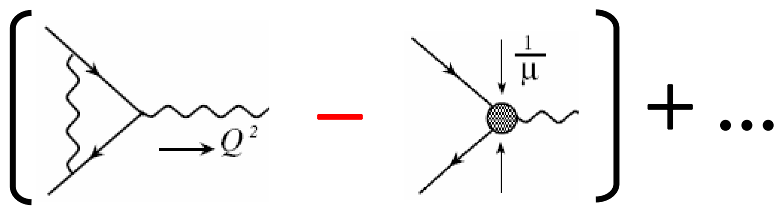
- ❖ combine the “high mass” states with LO

LO:



Renormalized coupling

NLO:



No UV divergence!

Renormalization = re-parameterization of the expansion parameter in perturbation theory

Renormalization Group

- Physical quantities can't depend on the renormalization scale - μ:

$$\mu^2 \frac{d}{d\mu^2}\sigma_{\text{phy}}\left(\frac{Q^2}{\mu^2}, g(\mu), \mu\right) = 0$$

$$\Longrightarrow \quad \sigma_{\text{phy}}(Q^2) = \sum_n \sigma^{(n)}(Q^2, \mu^2)\left(\frac{\alpha_s(\mu)}{2\pi}\right)^n$$

$$\alpha_s(\mu) = \frac{g^2(\mu)}{4\pi}$$

- The β-function:

$$\beta(g) = \mu\frac{\partial g(\mu)}{\partial \mu} = +g^3\frac{\beta_1}{16\pi^2} + O(g^5)$$

$$\beta_1 = -\frac{11}{3}N_c + \frac{4}{3}\frac{n_f}{2} < 0 \quad \text{for } n_f \le 6$$

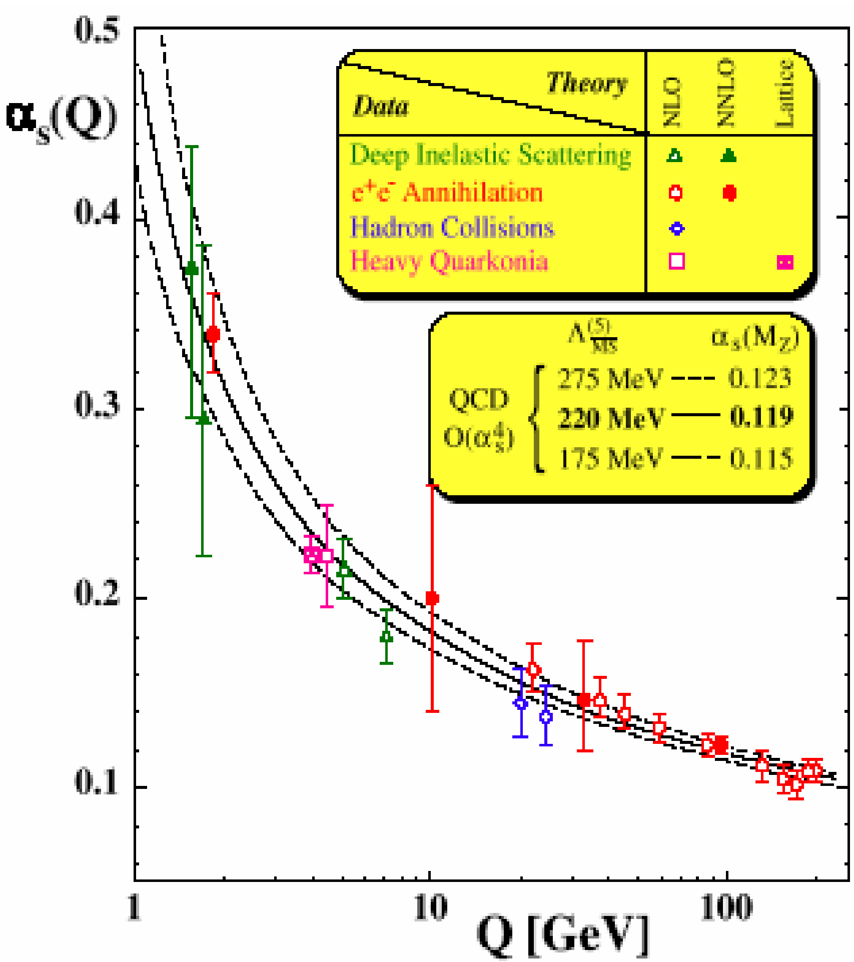
- QCD running coupling constant:

$$\alpha_s(\mu_2) = \frac{\alpha_s(\mu_1)}{1 - \frac{\beta_1}{4\pi}\alpha_s(\mu_1)\ell n\left(\frac{\mu_2^2}{\mu_1^2}\right)} \Rightarrow 0 \text{ as } \mu_2 \to \infty \quad \text{for } \beta_1 < 0$$

Asymptotic freedom

QCD running coupling constant

- Λ_{QCD}: $$\alpha_s(\mu_2) = \frac{\alpha_s(\mu_1)}{1 - \frac{\beta_1}{4\pi}\alpha_s(\mu_1)\ell n\left(\frac{\mu_2^2}{\mu_1^2}\right)} \equiv \frac{4\pi}{-\beta_1 \ell n\left(\frac{\mu_2^2}{\Lambda_{QCD}^2}\right)}$$



μ_2 and μ_1 not independent

Asymptotic Freedom ⇔ antiscreening

$$\text{QCD: } \frac{\partial \alpha_s(Q^2)}{\partial \ln Q^2} = \beta(\alpha_s) < 0$$

Compare

$$\text{QED: } \frac{\partial \alpha_{EM}(Q^2)}{\partial \ln Q^2} = \beta(\alpha_{EM}) > 0$$

D.Gross, F.Willczek, Phys.Rev.Lett 30, (1973)
H.Politzer, Phys.Rev.Lett. 30, (1973)

2004 Nobel Prize in Physics

Effective quark mass

- **Running quark mass:**

$$m(\mu_2) = m(\mu_1)\ \exp\left[-\int_{\mu_1}^{\mu_2} \frac{d\lambda}{\lambda}[1+\gamma_m(g(\lambda))] \right] \Rightarrow \ 0 \ \text{ as } \mu_2 \to \infty$$

Perturbation theory becomes a massless theory when $\mu \to \infty$

- **for light quarks, *u* and *d*, even *s*,** $m_{u \text{ and } d}(\mu) \ll \Lambda_{\text{QCD}}$
- **Choice of renormalization scale:** $\mu \sim Q$

QCD perturbation theory (Q>>Λ_{QCD})
is effectively a massless theory

Infrared Safety

- Infrared safety:

$$\sigma_{\text{phy}}\left(\frac{Q^2}{\mu^2},\alpha_s(\mu^2),\frac{m^2(\mu^2)}{\mu^2}\right) \Rightarrow \hat{\sigma}\left(\frac{Q^2}{\mu^2},\alpha_s(\mu^2)\right)+O\left[\left(\frac{m^2(\mu^2)}{\mu^2}\right)^{\kappa}\right]$$

Infrared safe = $\kappa > 0$

Asymptotic freedom is useful only for quantities that are infrared safe

Asymptotic freedom + Infrared safety = perturbative QCD

Foundation of perturbative QCD

- **Renormalization**
  - **QCD is renormalizable**

- **Asymptotic freedom**
  - **weaker interaction at a shorter distance**

- **Infrared safety**
  - **pQCD factorization and calculable short distance dynamics**

Summary

- QCD is a SU(3) color non-Abelian gauge theory of quark and gluon fields

- QCD perturbation theory works at high energy because of the asymptotic freedom

- QCD perturbation theory is effectively a massless theory – renormalization group equation for the parton mass

- Perturbative QCD calculations make sense only for infrared safe (IRS) quantities

Look for IR safe quantities